RANKING BY CONSENSUS USING ONE-SIDED DEA

by

JOHN PASQUALE RUGGIERI

Thesis submitted in conformity with the requirements
for the Degree of Doctor of Philosophy

Centre for Management of Technology & Entrepreneurship
Graduate Department of Mechanical and Industrial Engineering
University of Toronto

SGS FINAL

Abstract

Ranking by Consensus Using One-Sided DEA – Doctor of Philosophy,
John Pasquale Ruggieri, 2004, Department of Mechanical & Industrial Engineering,
University of Toronto.

The extension of Data Envelopment Analysis (DEA) developed in this thesis presents a new approach for calculating a common set of weights for use in the scoring and ranking of alternatives in a project selection problem setting. In contrast to the traditional approach where expert opinion is interpreted to derive weights for aggregating individual criterion scores of an alternative, a new development is introduced to derive these weights from empirical data, based on a set of new One-Sided DEA (OSD) models and a new Ranking by Consensus (RC) methodology.

The RC methodology was applied to four relevant problems: 1) Power Plant site selection, 2) Shortlisting Research Grant Proposals for a large Government Granting Agency 3) Capital City site selection and 4) Two-dimensional simulated data for illustration. The results offer new insights into the possibilities of novel tools which can be helpful in the decision making process by providing a mathematical ranking of alternatives based on a common set of weights.

This set of weights have been shown to represent the consensus opinion of the very candidates being evaluated, with the understanding that the candidates were free to selfishly select weights, with full knowledge of all other candidate's criteria scores, so as to make themselves appear as attractive as possible in the respective selection problem setting. The interpretation and implication of this common set of weights is explored and recommendations for extending the methodology in practice are made.

Table of Contents

List of Figures

List of Tables

Dedication

To Kydo™.

Acknowledgements

Thank you to Dr. J.C. Paradi for saving my life.

Thank you to Dr. I.B. Turksen, Dr. B. Benhabib & Dr. C-G Lee for sparing it.

Thank you to Dr. O. Despić & Dr. F.K. Tam for teaching me how to play.

Thank you to all the people at the CMTE for showing me the way.

Thank you to my wife Andrea and to my Family for loving me through the process.

Thank you to Dr. G.G. Hatch, R.R. Marshall, K. McEvoy & J. Raynes for inspiring me to aspire.

Thank you to NSERC (Grant # PGSB-222266-1999) and the Royal Bank of Canada, Canadian Imperial Bank of Commerce and TD Canada Trust for making it possible.

Thank you to the University of Toronto, her staff and faculty for keeping me smiling.

Glossary of Terms

Additive Model — A DEA model that shares the same envelopment surface as the BCC model but projects inefficient DMUs through stepwise movements along slack variables

Alternatives — Those decision outcomes, objects or entities to be ranked

Analytic Hierarchy Process (AHP) — MCDM technique based on the pair-wise comparison of relative importance of criteria for the elicitation of preference weights

Assurance region — A weight restriction technique in DEA based on fixing pair wise trade-offs between different inputs and/or outputs

BCC Model — A Variable Returns to Scale radial DEA model

Categorical Variables — Those variables in a DEA study allowing for the grouping of DMUs into sub-classes for relative efficiency evaluation

CCR Efficiency — A term relating to a DEA study based on the CCR model. A unit is deemed to be CCR efficient if its efficiency score is 1.0 and all slack variables are zero.

CCR model — Original DEA model for measuring relative efficiency of decision making units in a constant returns to scale setting

Cone ratio model — A weight restriction technique in DEA based on restricting weight selection to pre-defined fixed closed cones

Consensus — That to which the majority does not object

Consensus Weighting — A common set of weights used in a Selection Problem setting for evaluating and ranking alternatives

Criteria — Those qualitative and/or quantitative measures on which an alternative is evaluated in a Selection Problem setting

Criteria Weights — A measure of perceived importance attributed to specific criteria in a Selection Problem setting

Cross Efficiency — Cross-Efficiency is defined as the efficiency score computed for a particular DMU using the weights chosen by another DMU.

Data Envelopment Analysis (DEA) — A body of concepts and methodologies using mathematical programming approaches to establish an efficient frontier for a given set of DMUs

DEA Derived Weights	Weights calculated for each criteria found using a DEA approach
Decision Maker (DM)	A Decision Maker is that which is accountable for a decision. It can be a person, group, institution, corporation or country.
Decision Making Unit (DMU)	A unit whose efficiency is evaluated by DEA
Desirability index	An MCDM term corresponding to a single score used to measure the relative attractiveness of an Alternative to a DM
Efficiency Score	The weighted sum of a DMU's produced Outputs divided by the weighted sum of its consumed Inputs. It is a single score between 0 and 1 that is a relative measure of production efficiency for a DMU
Envelopment Form	The primal form of DEA models
GGA	Acronym for Government Granting Agency. An organization responsible for the dissemination of research funds
Group Decision Making	A subset of MCDM theory specifically focused on aggregating and eliciting group perspectives to gain consensus for the outcome of a decision process
Input Oriented	A DEA model that projects inefficient units to the frontier based on an equi-proportional reduction of all inputs
Inputs	Items consumed in a production process. Generally, those measures we would like to minimize.
Multi Criteria Decision Making (MCDM)	A body of concepts and methodologies for making decision in the presence of many conflicting criteria
Multi-attribute Utility Theory	MCDM technique based on evaluating utility value of individual criteria to a Decision Maker
Multiplicative Model	A DEA model with a piece-wise Cobb-Douglas non-linear frontier surface
Multiplier Form	The dual form of DEA models
Non-discretionary variables	Those inputs and/or outputs in a DEA study that are beyond the control of the management of a DMU
Non-parametric	Not constrained to any preconceived functional form. DEA is a non-parametric technique in that the production possibility set is derived from empirical data

Operations Research	The use of quantitative models to analyze and predict the behavior of systems that are influenced by human decisions
OSD	Short form for One-Sided DEA. A DEA model with only all Inputs or only all Outputs
OSD-CCR	A one-sided DEA model, based on the standard CCR formulation with only Outputs and one constant dummy input which calculates an optimal distribution of weights for an alternative so as to maximize its score
OSD-DA	A one-sided DEA model that calculates an optimal distribution of weights for an alternative so as to maximize its positive distance from the average
OSD-IP	A one-sided DEA model that calculates an optimal distribution of weights for an alternative so as to maximize its rank
Output Oriented	A DEA model that projects inefficient units to the frontier based on a equi-proportional expansion of all outputs
Outputs	Items produced in a production process. Generally, those measures we would like to maximize.
Outranking Methods	MCDM technique based on defining a set of alternatives that dominate all other alternatives
Parametric	Techniques based on preconceived values, prices or weights
Pareto efficiency	An allocation of resources is Pareto efficient if there is no way that some individual could be made better off without making any other individual worse off.
Productivity	The output of any production process, per unit of input.
Ranking	The process of putting things in order.
RC	Acronym for Ranking by Consensus. A methodology for ranking decision alternatives using criteria scores
RCM	Acronym for 'the Ranking by Consensus Methodology'
Selection Problem	Subset of Multi-Criteria Decision Making problems that involve the selection of a subset of possible alternatives to meet a particular goal
Standardized Ranking Space (SRS)	n-dimensional vector space for mapping alternatives based on n distinct criteria that are mapped and constrained to the full [0,1] scale

Translation invariance	Desirable property displayed by several DEA models allowing for the linear translation of inputs and/or outputs for handling negative and/or zero values
Unrestricted DEA	A DEA calculation without externally imposed weight restrictions
Weight Importance Derived Efficiency (WIDE) Score	The scores calculated by taking the dot product of the vector of actual *normalized* criteria values by the WIF vector resulting in a single score between 0 and 1
Weight Importance Factor (WIF)	Normalizing the values of the area under all weight importance graphs gives one vector called a Weight Importance Factor (WIF). The WIF will have as many elements as there are WIGs and simply presents the proportion, on average, that a DMU attributes to each corresponding criteria. The WIF is the link between OSD derived weights and MCDM expert elicited weights. The WIF vector will have elements that sum to 1 and represent a common set of weights
Weight restrictions	In DEA, the incorporation of a priori knowledge on the absolute or relative price of inputs and outputs
Weight Importance Graph (WIG)	An ordered distribution of the weight assigned by *each* DMU to *one* criterion in an RC study
ε	Non-archimedian constant corresponding to smallest allowed value greater than zero
θ	Symbol for efficiency score from CCR input oriented primal formulation
ϕ	Symbol for efficiency score from CCR output oriented primal formulation
Ω	Objective function value from OSD-IP formulation corresponding to the number of alternatives that rank above the alternative being analysed
Φ	Objective function value from OSD-DA formulation corresponding to a measure of distance between the alternative being evaluated and the average alternative
λ_p	Non-negative scalar in the Primal LP form of DEA that, if solved to be positive, indicates that the DMU_p is a member of the peer group of the DMU being analysed
$\boldsymbol{u_r}$	Vector of weights assigned to outputs values
$\boldsymbol{x_{ij}}$	Vector of input values for DMU j

$\boldsymbol{x_{io}}$	Vector of input values for DMU being analysed
$\mathbf{Y}$	Matrix of output values
$\boldsymbol{y_{rj}}$	Vector of output values for DMU j
$\boldsymbol{y_{ro}}$	Vector of output values for DMU being analysed
$\boldsymbol{v_i}$	Vector of weights assigned to input values
s	Total number of outputs for each DMU
$\boldsymbol{E_e}$	Engineering definition of efficiency defined as Outputs/MPY and constrained to [0,1]
$\boldsymbol{E_s}$	Systems definition of efficiency defined as Output/Inputs and constrained to [0,1]
$\boldsymbol{g_o}$	Symbol for efficiency score from CCR output oriented dual formulation
m	Total number of inputs for each DMU
MPY	Maximum possible output
n	Total number of DMUs in a DEA study
$\boldsymbol{h_o}$	Symbol for efficiency score from OSD-CCR formulation
u_r^*	An optimal set of weights calculated using an OSD model

1.0 INTRODUCTION

"Sometimes a scream is better than a thesis." - *Ralph Waldo Emerson*

Decisions, Decisions, Decisions… the three most important activities of corporate executives and politicians alike. The dollar consequences can be enormous and their quality and timeliness can make or break any organization. This thesis begins by considering a project selection problem where the goal is to assist a Decision Maker (DM) in selecting a subset of available alternative proposals for allocation of more resources; whether that be more time, more money or more information.

Inspection of the generic selection problem led to the realization that sorting alternatives into two distinct groups (those chosen and those not chosen) was equivalent to the problem of ranking all of the alternatives from most preferred to least preferred and cutting at some level. Hence, while we initiated this research in the domain of selection problems, we ended up creating something new in the domain of ranking.

Furthermore, upon application of the models and methods developed here we soon discovered that our approach was entirely new and despite its simplicity, has significant potential well beyond the specific problems addressed here. The last Section offers a glimpse of some of the possibilities available to future developers who can recognize the opportunities at hand.

This thesis documents seven years of research, both theoretical and applied, to the problem of improving the process of screening investment alternatives.

On the practical side, we investigate four relevant datasets, all utilizing screening methodologies that score alternatives on a number of subjective and/or objective criteria, weigh those criteria scores based on expert opinion, and then use the resultant product of the two to create an overall single score which is used to rank and hence filter the alternatives. One dataset involved the selection of a candidate site for a power plant, the second dataset involved the dissemination of government

grants to research institutions, the third dataset involves the analysis of candidate sites for a new capital city and the fourth dataset is simulated and was created randomly to illustrate our contributions. In these cases, the methodology and models developed as part of this research was applied, yielding new insight into the respective problems and a new ranking of the decision alternatives, based on the consensus weighting of selection criteria.

On the theoretical side, the problem was vexing and required a new approach. The literature reviewed led us to speculate that one could gain new insight into selection problems of this nature if one were to have access to the perspectives of the alternatives being evaluated. Relating this back to the types of selection problems we were originally focused on, we can see that this perspective may be inferred by examining those criteria weights that would be chosen by each of those alternatives (grant applications or business plans for example) if they were free to selfishly choose weights (with full knowledge of all other alternative's criteria scores) so as to make themselves appear as attractive as possible in the respective problem setting.

The significance of this theoretical advance lies in the fact that, in many important situations, leaders and administrators must provide rationale and justification for their decisions to the constituency affected by such decisions. The methodology and models developed in this thesis provide a new mathematical justification for ranking alternatives and more importantly, for the first time, provides a mechanism for incorporating the derived opinion of the constituency, directly into the decision making process.

1.1 ASSUMPTIONS & LIMITATIONS

This thesis presents a new methodology and new models for the mathematical ranking of decision alternatives. Before application, a number of assumptions have been made as to the soundness and appropriateness of such methods. It is always the case that there exists an inherent uncertainty in obtained results derived from methods based on the mathematical treatment of subjectively collected and measured values. This uncertainty can arise from noise in the system, incomplete information,

bias in the measurement of values, limitations on the accuracy of numerical methods and limits in the precision of the computers and software used to solve the resultant systems of equations. It must be stated that strict reliance on the numerical values derived using the tools presented herein is not only unscientific, but also potentially dangerous and, under certain circumstances, counter productive. One must always view the results obtained from such an analysis with a measure of skepticism and some detachment. The tools and methods presented here are intended to aid a decision maker by structuring their output in such a way as to assist in the sound discussion and analysis of large problems. Our methods are not autonomous systems for making decisions. The responsibility of choosing the actual alternative outcomes will always reside with the decision maker – therefore sound human judgment must not only be taken in the application of these methods, but also in the acceptance or rejection of the results.

Furthermore, the models and methods presented in this thesis assume the most basic linear decision function – the linear aggregation of independent criteria. While this model helps to make the understanding and discussion of decisions more pragmatic, it by no means can come close to incorporating or modeling all of the complex interactions and considerations that are truly involved in making important decisions. More advanced non-linear or quasi-linear approaches do exist and need to be explored but we are always faced with the trade-off of improving accuracy at the cost of adding complexity. Therefore, we sincerely trust that these methods will be used with caution and proper due diligence by those experts who not only understand the limitations and complexities of such an approach but who are also willing and able to accept the responsibility that arises from their use.

This thesis presents three new models for calculating criteria weights, in a selection problem setting, and a methodology for using them. These three models are not the only models that exist or could be created for this purpose. New models and combinations of models will give dramatically different results and rankings of decision alternatives. Also, the necessary work to devise a way to place confidence in the results obtained from the application of our methods and to rigorously define

thresholds of distinguishability have yet to be performed. This may make the presentation and discussion of our results and conclusions seem more certain than they actually are. The semantics of interpreting mathematical results and putting language around any conclusion is problematic and naturally this work is prone to this limitation. We strongly suggest a thorough specification of the application context before the use of these methods and encourage deriving consensus among the decision makers as to which model or models are the most appropriate.

Of course, common sense must prevail in all decision situations and we acknowledge that the application of our method and models can be inappropriate in certain settings. As an example, in situations where a parent must make decisions for a child in dangerous environments, especially when time is critical, these methods may be unsafe and we humbly defer to good judgment and instinct when the decision maker knows the best decision alternative.

1.2 PROBLEM DEFINITION

Given the assumptions and limitations discussed in Section 1.1, simply stated, the theoretical problem we were faced with can be summarized as: **"Given a finite set of alternatives, evaluated on a finite set of criteria, rank the alternatives."**

Specification of this problem led us to the broad research areas of Data Envelopment Analysis and Multi Criteria Decision Making and a need to marry the two.

Data Envelopment Analysis (DEA) is one of the recognized approaches for analysing Multi Criteria Decision Making (MCDM) type problems when expert opinion is not available [STEW94]. In a typical MCDM setting, different methodologies can be employed to derive a set of criteria weights used for evaluating and ranking alternatives. **The choice of weights is very important as different weights can lead to dramatically different rankings.** These weights are usually derived using expert opinion or decision maker preference data. In DEA literature however, these weights are calculated individually for each alternative in such a way that makes each alternative appear in the 'best possible light'.

The use of DEA for MCDM problems has been gaining significant attention since Stewart added the technique to his survey of MCDM techniques in 1994 [STEW94]. Stewart recognized the unique characteristics of DEA as an additional tool a decision maker could use when analysing MCDM problems. Since then, significant extensions to DEA theory, for example Cross-Efficiencies, published by T. Sexton in 1986, have been used in an attempt to gain new insight into these types of problems.

However, the literature still identifies two major issues with using DEA on MCDM problems, which this research addresses successfully.

The first problem is the difficulty in categorizing decision making criteria into Inputs and Outputs as required for a traditional DEA analysis. The second problem is DEA's reliance on 'strange ratios' when calculating efficiency scores. These two problems are further explained and their history explored in the literature review.

This research avoids the problems reported by introducing a spectrum of new one-sided DEA models for MCDM problems. Building on these models and the assumptions required for their application, a new Ranking by Consensus (RC) methodology was created. This new methodology formulates such decision problems in such a way that the result from its application is a vector of weights that has a new and intuitive meaning corresponding to the consensus of how the alternatives themselves would want to be evaluated.

In a typical MCDM problem, an expert may assign weights to each criterion to score and rank alternatives. This new methodology accomplishes the same goal without requiring a priori human experts' opinion, represented as a vector of weights, by using advanced mathematical programming techniques to calculate these weights from each alternative's perspective and aggregate them for consensus. This process provides a decision maker with new insight into such problems and provides a common framework for comparing expert derived weights and mathematically derived weights. The ultimate purpose of which is to provide feedback to decision makers on the decision process and in particular, to facilitate communication of the relative importance of decision criteria from multiple perspectives.

1.3 MOTIVATION

The motivation for this work came from the desire to create a new, theoretically sound approach, which could be used to assist a large Government Granting Agency (GGA) in the screening of Research Grant Proposals (RGPs). The GGA funds a number of programs which provide grants to research institutions for R&D related projects. Each of the research institutions requesting a grant must complete an assessment form of nine questions, which have been designed to match the values and objectives of the GGA. All of the nine questions are assigned an integer response in the range of one to four (the higher, the more desirable). The current system then weighs each of the criteria and an aggregate score is used to rank and screen projects for further due diligence.

Both Federal and Provincial governments allocate many millions of dollars per year for supporting various programs such as research and infrastructure development, with the assistance of computer-based tools for scoring and ranking of applicants. One way to deal with the generic selection problem is to first rank the decision alternatives. Ranking has the benefit of allowing a Decision Maker to select the best, sift through the difficult middle group and quickly discard those that do not merit support.

The linear programming technique, Data Envelopment Analysis, was used as the starting point in the development of a new approach for a generalized selection process and new ranking methodology. The advantage of DEA is that it allows for the simultaneous comparison of all decision alternatives across multiple criteria without requiring a priori human experts' opinion or a preconceived parametric model. This work managed to extract new information that is inherent in the empirical data by interpreting the relative differences between and across actual measured data values.

While attempting to apply this new theoretical development to the selection of proposals we were challenged by an interesting dilemma. The privacy policy of the GGA restricted us from taking a deep look into the specifics of each proposal.

Moreover, time and resource limitations prevented the in-depth interviewing of experts about particular proposals or projects and it was not practical to follow up on individual proposals to evaluate actual outcomes. For our purposes, this was not a limitation as our intention was to focus on the mathematical modeling of such decisions and improve that process. Given the interesting dataset and restrictions we were faced with, this lead us to the objective of combining, in a novel way, two distinct, yet complementary Management Science disciplines, Multi-Criteria Decision Making (MCDM) and Data Envelopment Analysis (DEA).

This was accomplished by creating a new way to model MCDM problems in DEA, based on a Standardized Ranking Space (SRS), that provides a framework for comparing expert derived weights from MCDM with calculated weights using DEA approaches. Based on this modeling technique, we were then able to create a new methodology for calculating the consensus weighting of selection criteria thereby enabling the consensus ranking of decision alternatives.

1.4 THE SETTING

While this research was conducted in the context of working to improve the current tools and methodologies used for the dissemination of Research and Development Grants, given certain assumptions and limitations, it may also be applicable and adaptable to those situation where the aggregation of many measures into a single score for ranking is needed.

The process of ranking is pervasive in our lives. We see it everywhere. It is the central theme behind sports and games, a persistent problem in the financial services industry and a sensitive issue for governments and governance. Why? Because ranking is the process of putting things in order. This problem is fraught with difficulties irrespective of how it is approached. Whether those things which we are trying to order are objects or people, ideas or approaches, companies or countries, we are always faced with the difficult task of justifying the final rank order. This

becomes even more problematic when the person or entity to which the final rank order must be justified to is one of the persons or entities that is being ranked.

Take for example the problem of ranking the medal winners from an Olympic Games competition. Each country competes in many events to try to place first, but sometimes they place second and other times they place third, winning them a gold, silver or bronze medal respectively. How then, do we rank all of the countries across all of the events? Do we rank based on total number of medals, paying no attention to the individual number of gold or silver or bronze medals – or – do we rank based on gold medals first, then by silver, then by bronze? If we take the first approach, then a country with one bronze medal would tie in rank with another country with one gold medal or even more counter intuitively, a country with two bronze medals would rank higher than another with only one gold medal.

Looking at the problem from the other perspective however is not much better. If we rank first by gold, then silver then bronze we could end up in the counter-intuitive scenario where a country that has fifteen bronze medals ranks below a country with one gold medal. Clearly, this is not fair. What is needed is a way to value or weigh the count of gold medals in terms of silver medals, and silver medals in terms of bronze medals and finally bronze medals in terms of gold medals. But what weightings do we use? Is a gold medal five times more valuable than a silver medal – or maybe three times? The final ranking depends on the distribution of weights across the different types of medals. Therefore, in this context, the final say as to the rank order of countries is completely dependent on the final weights chosen.

Who should have the final say in the final weights chosen for such a ranking problem? There is no clear answer to this question and the International Olympic Committee avoids the issue by not presenting a final ranking of countries, only tables of medal counts. This is understandable considering the delicate nature of the problem and the fact that the IOC is concerned with celebrating human performance – not ranking.

What we are proposing in this thesis is a methodology that allows the countries themselves to come to a consensus on what the final weights should be in terms of the relative importance of gold, silver and bronze medals. Once we have agreement there, the final ranking of the countries is easy to calculate and defend. This solution seems simple enough but we have just replaced one problem with another, which is how do we get all the countries to agree on one common set of weights? Taken further, the approach can then be applied to other sporting venues like individual player ranking, ranking of draft picks and overall team standings. The methodology and models introduced in this thesis could present a compelling answer to these and many other similarly vexing questions in a wide range of application domains.

Moving into the world of financial services, we see the problem of ranking surface when assessing credit worthiness, risk tolerance and portfolio selection – to name just a few. Each of these applications involves the simultaneous consideration of many conflicting measures in an attempt to rank the alternatives. The standard way to deal with this problem involves the creation of a single score or index that combines the disparate measures to allow for comparison. Here, we present a new approach for calculating this index which gives different yet defendable results.

Governments face the same issue. Governance relies on the appropriate ranking of decision alternatives and more importantly; good governance relies on the appropriate incorporation of many perspectives into such decisions. Reaching consensus is the major hurdle as we see it and here we present a new method to achieve that goal.

1.5 OBJECTIVES OF RESEARCH

Our objective in conducting this research was to investigate the feasibility of using DEA to aid decision makers in their own setting. Given its non-parametric approach, ability to handle multiple inputs and outputs of different measurement units and previous research showing unique insight into similar problems, DEA provided a promising solution.

More specifically this research intends to:

Create a New Methodology for the Ranking of DMUs from a large group

- Create a new approach, based on DEA, for mathematically ranking DMUs from a list of competing candidates

- Develop a new extension to the theoretical proposition of using DEA as a pre-processor in MCDM problems where selection of a number of alternatives out of a larger population is required and expert opinion is not available.

- Demonstrate these theories by applying them to real-life data

Extend the use of DEA as part of a new a Multi Criteria Decision Making (MCDM) approach

- Combine MCDM and DEA theories for the selection of criteria weights and propose a theoretical framework for understanding DEA derived weights.

- Explore the relationship between DEA derived weights and Expert Opinion in MCDM. Specifically, looking at the weights defined using the new DEA based models and justifying the analogy of those weights as reflecting the consensus of how DMUs want to be evaluated.

- Develop a new methodology for applying DEA to MCDM problems that overcomes the recognized pitfalls as reported in the literature.

1.6 OUTLINE

Section 2 reviews the relevant literature as it relates to decision making under multiple, often conflicting criteria, with particular focus on the use of Data Envelopment Analysis in this setting. Section 3 is a technical review of the field of DEA, in the traditional area of efficiency analysis. The major contribution of this work, a spectrum of One-Sided DEA (OSD) models and the RC Methodology (RCM) are presented in Section 4 including derivations and extensions. Applications to three real world datasets, and one simulated dataset, are presented in Section 5. Conclusions and recommendations for future work complete the document in Section 6 while Section 7 offers a glimpse of what the future might hold for this new ranking technology.

2.0 LITERATURE REVIEW

"If I have seen further it is by standing on the shoulders of Giants" – Sir Isaac Newton

This section is intended to give the reader an overview of the literature as it pertains to decision making when multiple, and possibly conflicting, criteria are to be considered. The body of concepts and methodologies has many names in the literature 1) Multi-Criteria Decision Making (MCDM), 2) Multi-Criteria Decision Analysis (MCDA), and 3) Multi-Attribute Decision Making (MADM) to name a few. Throughout this work we will limit ourselves to the MCDM terminology.

Alternatives: the decisions, objects or entities to be ranked
Criteria: dimensions on which the alternatives are measured
Weights: the importance placed on criteria
Value: the measured amount of a particular criterion
Score: a single evaluation number of an alternative
Rank: the position in the ordering of alternatives

The goal of this section is to start at the very beginning, first defining and differentiating the three types of decisions that can be made and then outlining a structured approach for rationally analyzing these types of decision problems. Three alternative approaches for formulating decision problems and aiding in decision-making, namely Multi-Attribute Utility Theory (MAUT), Analytic Hierarchy Process (AHP) and the Outranking Method are then described.

Due to the nature of the problems this thesis is focused on addressing – selection problems in a group environment – group decision-making theory is briefly covered, specifically focusing on the theoretical underpinnings involved in the aggregation of individual perspectives and preferences.

From there, we follow with a more in-depth look at the use of Data Envelopment Analysis as an alternative approach for formulating MCDM problems, highlighting some of its limitations and focusing on its use in calculating preference weights for selection criteria. This section concludes with the identification of two referenced

problems associated with using DEA in an MCDM environment, which this work is positioned to resolve.

2.1 DECISION MAKING

According to Roy, three major decision problem formulations exist: choice, sorting and ranking [ROY96]. For any of these problem formulations, a solution must be found with respect to the preferences of a single (or a group of) decision maker(s). Various approaches and supporting software tools have been proposed to aid in this task. The most important, and the ones surveyed here, are Multi Attribute Utility Theory (MAUT) [KEEN76], the Analytic Hierarchy Process (AHP) [SAAT80] and the Outranking methods [ROY90].

Let us take a step back and look closely at the statement made by Roy concerning the three types of decisions that can be made: 1) Choice, 2) Sort and 3) Rank. At first glance, it may appear that each of these distinctions are simply different descriptions of the same thing, a 'decision', but on closer examination, it becomes clear that they truly are three distinct subclasses of what we generalize to be a 'decision'.

To understand the distinction between these three different types of decisions, we consider an extremely simple example, starting with a jar of buttons. Consider the case where one is presented with a jar of randomly collected buttons, containing several hundred samples, varying in size, shape, colour, number of holes, and material. In this rudimentary example, each button corresponds to an alternative and the five characteristics correspond to five selection criteria. Now imagine that we empty the jar onto a table. We are now in a position to understand the distinction between the three types of decision problems. A 'choice' decision is characterized by the concept of 'preference', and in this case would correspond to choosing your favourite three buttons, for example. In general, a choice involves selecting a subset of all possible alternatives. A sorting decision is quite different. When sorting, the decision is whether to sort based on any one of the five criteria – do you sort by

colour, or size or shape? Also, sorting can be nested. One could sort by shape, and then sort each sub group by size. And finally, rank. Ranking implies one continuous ordering of all alternatives, from best to worst. How would one rank the alternatives in this example? It becomes a function of both preference *and* criteria values – much more difficult than the previous two.

So, to summarize, Choice implies Preferences, Sorting implies Grouping and finally Ranking implies Order.

Why is it necessary to distinguish between the three types of decisions and to relate the problem back to buttons? Well, let us consider the original problem setting of this thesis and map the alternatives back to Roy's framework. In the case of the GGA, we have 200 grant applications scored on 9 criteria. The goal of the GGA is to choose a subset of these alternatives to fund. However, due to the sensitive political nature of the decisions to be made, individual preferences need to be minimized if not removed completely, therefore it is not a *choice* decision. Also, in an ideal world, the alternatives would be ranked, from best to worst, allowing the GGA to fund all of those projects until the budget has been exhausted. This is fraught with problems in that the ranking must be defendable to the alternatives (applicants) that are requesting a grant. Therefore, this implies that this is a *sorting* decision, where the goal is to sort the alternatives into two groups, those to fund and those not to fund. This lends itself nicely to the methods of DEA in that, as presented in Section 2.4.6, DEA separates DMUs into two groups, those that are *efficient* and those that are *inefficient*.

Once we recognize the type of decision to be made, we are now free to use a structured methodology for analyzing and presenting a decision, the alternatives and rationale to a Decision Maker.

2.1.1 Structured Decision Making

The major benefit of taking a structured approach to decision making is to allow for multiple decision makers to be involved in the decision. For those decisions where

the consequences of a decision has a large impact (i.e. on society or the environment), it becomes of paramount importance to include the perspectives of as many experts as possible and to ensure that all practical aspects of a decision have been properly considered and measured in the process.

The use of mathematical models for problem structuring and modeling is especially advantageous as it provides a scientifically sound structure upon which to map said problems that are universally understandable and defendable. Also, modeling in this way provides for new insights into problems while enabling new solutions that would otherwise be too cumbersome to derive through inspection alone. The drawback to such an approach is that the mathematics involved in the more sophisticated tools tends to be too complex for the average person, thus limiting those who can participate in a decision making process or adding an extra layer of translation through the use of analysts who must translate from expert to model and back from model to expert. To be fair, these issues are becoming less of a hurdle in real world situations with the advent of new tools and computer based models. It is hoped that the work contained in this thesis is a further step in the direction of making sophisticated, mathematically sound decision aids more transparent and thus more accessible to the everyday decision maker.

Olson et. al identify the four major steps in decision analysis as [OLSO01]:

1. Problem Structuring
2. Decision Strategy Development
3. Elicitation of Preference Information
4. Analysis of the Result

Problem structuring refers to the identification of all relevant alternatives and criteria. The standard way of tackling this step is to identify as many alternatives as possible and then to attempt to distinguish between them to define the criteria. Take for example the decision to select a movie to see. The alternatives would be the movies that are available. Comparing two movies, we might find that one is 90 minutes long and the other 180 minutes. This distinction allows us to see that one criterion for choosing could be length of movie – for example.

Once the criteria have been identified, a decision strategy is chosen where decision strategy can be either compensatory (which implies a finite tradeoff among each pair of criteria) or non-compensatory. Compensatory can be directly cardinal or take an outranking approach.

Decision Aid	Weight Elicitation	Score Elicitation
MAUT	Tradeoff	Utility Scores
AHP	Ratio pair wise comparisons	Ratio pair wise comparison
Outranking	Direct	Concordance, discordance

Table 1 – Weight and Score Elicitation for Three Decision Aids

Next is the elicitation of preference information from the DM. Table 1, reproduced from [OLSO01] summarizes traditional weight elicitation techniques.

Using one of the three approaches listed in Table 1, elicitation of weights is accomplished through interaction with the DM, possibly iterating throughout the process. Aggregating and interpreting expert responses allows for DM preferences to be injected into the appropriate model and eventually leads to a calculated rank ordering (or partial rank ordering) of alternatives to aid in the decision process.

While Buchanan states decision makers need only a few cues in order to make consistently good judgments, the use of structured approaches enables the evaluation of many more alternatives, across many more criteria and provides justification for reducing decisions to a smaller number of criteria when appropriate [BUCH01].

2.2 MULTI CRITERIA DECISION MAKING (MCDM) METHODS

Multi Criteria Decision Making (MCDM) is a powerful management science discipline with a rich history and solid foundation. A comprehensive bibliography on the subject can be found at www.lamsade.dauphine.fr/mcda/biblio and lists more than 3000 authors and contains bibliography entries dating back to as early as 1736. Hwang et. al, Zeleny and Yoon et. al provide good introductions to MCDM concepts and a review of conventional MCDM methods [HWAN81][ZELE82][YOON95].

While a full review of the discipline is beyond the scope of this work, the relevant areas needed for this thesis are presented below.

2.2.1 Multi Attribute Utility Theory (MAUT)

Multi Attribute Utility Theory is a structured methodology designed to handle the tradeoffs among multiple objectives. One of the first applications of MAUT involved a study of alternative locations for a new airport in Mexico City in the early 1970s. The factors that were considered included cost, capacity, access time to the airport, safety, social disruption and noise pollution.

Utility theory is a systematic approach for quantifying an individual's preferences. It is used to rescale a numerical value on some measure of interest onto a 0-1 scale with 0 representing the worst preference and 1 the best. This allows for the direct comparison of many different measures. Resulting in a rank ordered evaluation of alternatives that reflects a DM's preferences.

A MAUT study starts with the establishment of a value function $v_k(f_k(a_t))$ that is normalized to the interval [0,1], where the best score on each criterion gets the utility value $v_k = 1$, and the worst one gets $v_k = 0$. Each alternative is then assigned an overall utility value based on [GELD02]:

$$v(a_t) = \sum_{k=1}^{K} w_k \cdot v_k(f_k(a_t)) \quad with\, w_k \geq 0 \; and \sum_{k=1}^{K} w_k = 1 \tag{2.1}$$

An analogous situation arises when individuals, college sports teams, MBA degree programs, or even hospitals are ranked in terms of their performance on multiple disparate measures [MERR02]. Another example is the Bowl Coalition Series (BCS) in college football that attempts to identify the two best college football teams in the United States to play in a national championship bowl game [MAIS98]. This process has reduced but not eliminated the annual end of the year arguments as to which college should be crowned national champion.

Early applications of MAUT focused on public sector decisions and public policy issues. These decisions not only have multiple objectives, they also often involve multiple stakeholders that are affected in different ways by a decision. Many power plant related decisions were made using MAUT. The military is also a leading user of this technique.

The application of MAUT requires the specification of weights by experts. The main idea behind MAUT is the ability to map measures on criteria to the *value* received by a DM if that alternative is chosen. The functional form for mapping is not standard and depends on the preferences of the DM and the specifics of the criteria being measured.

The Analytic Hierarchy Process (AHP) is a direct competitor to MAUT as an efficient technique for rank ordering alternatives.

2.2.2 Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process (AHP) is a powerful and flexible decision making process to help people set priorities and make better decisions when both qualitative and quantitative aspects of a decision need to be considered. By reducing complex decisions to a series of one-on-one comparisons, then synthesizing the results, AHP not only helps decision makers arrive at the better decisions, but also provides a clear rationale of the decision process. Designed to reflect the way people think, AHP was developed in the 1970's by Dr. Thomas Saaty, and continues to be among the highly regarded and widely used decision-making approaches [SAAT80].

AHP involves structuring a problem from a primary objective to secondary levels of objectives. Once these hierarchies have been established, a pair wise comparison matrix of each element within each level is constructed. Participants can then weigh each element against each other element within each level, each level is related to the levels above and below it, and all levels are tied together mathematically. The result is a clear priority statement of an individual or group.

The following example is taken from [TRIA00] and illustrates the final ranking of three alternatives based on the relative pair wise comparison derived decision matrix.

Given the following three alternatives (A1, A2, A3), evaluated on four criteria (C1, C2, C3, C4):

Criteria				
	C1	C2	C3	C4
Alts.	0.20	0.15	0.40	0.25
A1	25/65	20/55	15/65	30/65
A2	10/65	30/55	20/65	30/65
A3	30/65	5/55	30/65	5/65

Table 2 – AHP Sample Decision Matrix

The AHP score for each alternative is found using Equation (2.2) resulting in AHP scores of 0.34 for A1, 0.35 for A2 and 0.31 for A3.

$$A^{*}_{AHP-score} = \max \sum_{j=1}^{n} a_{ij} w_j \qquad i = 1, \ldots, m \tag{2.2}$$

This leads to the overall ranking of A2>A1>A3.

2.2.3 Outranking Approaches

Outranking approaches originated in France from the work of Bernard Roy et. al in the mid-1960s and has continued to be applied and extended since that time.

The main outranking methods assume data availability similar to that required for other MCDM techniques. That is, they require alternatives to be specified, their performance to be assessed on a number of criteria and for weights to be expressed indicating the relative importance of the different criteria.

Outranking may be defined as follows [ROY96]:

Option A outranks Option B if, given what is understood of the decision maker's preferences, the quality of the evaluation of the options and the context of the problem, there are enough arguments to decide that A is at least as good as B, while there is no overwhelming reason to refute that statement.

Thus outranking is defined fundamentally at the level of pair wise comparison between every pair of options/alternatives being considered.

Based on this rather general idea, a series of procedures have been developed to operationalise outranking as a way of supporting multi-criteria decision-making. Typically, the process involves two phases. First, a precise way of determining whether one option outranks another must be specified. Secondly, it is necessary to determine how all the pair wise outranking assessments can be combined to suggest an overall preference ranking among the options.

There are many outranking methods that have been built on this basic premise. Here we will describe the most basic, developed by Roy called the ELECTRE I method [ROY99].

ELECTRE I

ELECTRE I is centered on identifying dominance relations. It works by locating a subset of alternatives, E, such that any alternative not in the subset is outranked by at least one member from the subset. The goal is to make the set E as small as possible and for it to act as a shortlist, within which a good compromise option should be found.

The process is initiated by first defining what are termed the concordance and discordance indices. The concordance index, $c(i,j)$, can be calculated for every ordered pair of options (i,j) simply as the sum of all the expert assigned weights for those criteria where option i scores at least as highly as option j.

The discordance index, $d(i,j)$, is a little more complex. If option i performs better than option j on all criteria, the discordance index is zero. If not, then for each criterion where j outperforms i, the ratio is calculated between the difference in performance level between j and i and the maximum observed difference in score on the criterion concerned between any pair of alternatives in the set being considered. This ratio (which must lie between zero and one) becomes the discordance index.

Defined in this way, the discordance index is only of real value in the later stages of the analysis if the criteria are roughly of equal importance. It is the discordance index that captures the notion of an alternative's unacceptability if it finds an outlying poor performer, even in just one dimension.

After the concordance and discordance indices have been found, the two are combined to define thresholds. To bring the two sets of n(n - 1) indices together for all n options being considered, the next phase is to define a (relatively large) concordance threshold, c*, and a (relatively low) discordance threshold, d*.

An alternative then outranks another alternative overall if its concordance index lies above the chosen threshold value and its discordance index lies below the threshold value. The set of all decision alternatives that outrank at least one other option and are themselves not outranked contains the promising solutions for the problem. If the set is too small, possibly an empty set, it can be expanded by changing the concordance and/or discordance thresholds. Similarly, if the set is too big, it can be made smaller in a similar manner.

The next step is the assessment of alternatives. Outranking methods tend to make fewer assumptions about the nature of the underlying process that produces preferences. It leaves more of the process of finalizing choice to the decision-maker through fine-tuning in terms of items like the concordance and discordance thresholds.

Outranking approaches recognise that decision alternatives, which perform relatively poorly on particular dimensions, may be hard to implement in practice. Overall, outranking is a more interactive process between decision maker and model than other approaches.

2.3 GROUP DECISION MAKING

While this work is not directly concerned with the topic of group decision making in a traditional sense, making the connection is a natural one. Considering each DMU

to be an expert in their own evaluation, leads to the question of how to aggregate all *n* DMUs determination of their own optimal weights.

2.3.1 Aggregating Group Perspectives

Borrowing from the pioneering work of Saaty, in [SAAT99], four conditions are identified that must be met to allow for the aggregation of individual judgments.

1. **Separability** condition (S): $f(x_1,x_2,\ldots,x_n)=g(x_1)g(x_2)\ldots g(x_n)$ for all x in P. This means that the influences of the individual judgments can be separated.

2. **Unanimity** condition (U): $f(x,x,\ldots,x)=x$ for all x in P. This means that if all individuals give the same judgment x, that judgment should also be the synthesized judgment

3. **Homogeneity** condition (H): $f(ux_1,ux_2,\ldots ux_n) = uf(x_1,x_2,\ldots x_n)$ where $u>0$ and x_k, ux_k $(k=1,2\ldots,n)$ all in P. This means that for ratio judgments if all individuals judge a ratio u times as large as another ratio, then the synthesized judgment should also be u times as large.

4. **Power** conditions (P_p): $f(x_1^p,x_2^p,\ldots x_n^p)=f^p(x_1,x_2,\ldots,x_n)$. This means that for P_2 example, if the k^{th} individual judges the length of a side of a square to be x_k, the synthesized judgment on the area of that square will be given by the square of the synthesized judgment on the length of its side.

The fourth condition has a special case for when p=-1, i.e. ($R=P_{-1}$): $f(1/x_1, 1/x_2,\ldots,1/x_n)=1/f(x_1,x_2,\ldots,x_n)$. **Ratio** condition (R) is of particular importance in ratio judgments. It means that the synthesized value of the reciprocal of the individual judgments should be the reciprocal of the synthesized value of the original judgments.

Theorem: The general separable (S) synthesizing functions satisfying the unanimity (U) and homogeneity (H) conditions are the geometric mean and the root mean power. If moreover the reciprocal property (R) is assumed even for a single n-tuple

$(x_1,x_2,\ldots x_n)$ of the judgments of n individuals, where not all x_k are equal, then only the geometric mean satisfies all the above conditions.

Abstracting to the case where some individuals are given more influence in their vote (either because they are more knowledgeable or more powerful), we now have the weighted separable condition, where the different weight is incorporated in the function g( ). In this case, the conditions and theorems from above still apply.

For our purposes and using Saaty's rationale, we can soundly conclude that the group perspective can be inferred and calculated by taking the weighted geometric mean of the individual weight vectors that are found by solving the representative linear programs. This, combined with the fact that each weight vector is calculated such that each DMU achieves its highest possible score (while maintaining feasibility) forms the basis for consensus weighting of the selection criteria.

2.4 Data Envelopment Analysis as an MCDM Tool

Data Envelopment Analysis (DEA) is a body of concepts, methods and models that have their roots in productivity/efficiency analysis. Section 3.0 provides a survey and summary of DEA in this setting. Here however, our aim is to survey the use of DEA as a tool for working with MCDM problems, not an area that was originally intended, but an area that has been receiving more attention in the literature over the last 10 years and an area that shows considerable promise.

As presented previously in this section, MCDM techniques require interaction with experts to elicit or calculate weights, which are used to present results. As will be seen, DEA is capable of incorporating expert opinion and/or decision maker preference data when required but, as is often the case, that information is not available. The exciting and interesting characteristic of DEA in this setting is that it can still be applied to MCDM problems, without external weighting procedures, by allowing each of the DMUs to evaluate themselves as well as their peers. Said another way, DEA uses properties of optimization embedded in linear programming

techniques to derive weights from empirical data, as opposed to eliciting those weights from experts. This is the property we wish to exploit.

Standard methods for using DEA for this type of analysis would involve codifying expert opinion and including it in the DEA model through weight restrictions and/or assurance regions. These methods require direct interaction with experts and are subject to all the biases and inherent difficulties involved in extracting and modeling expert opinion. Models that do not take into account expert opinion rely on the standard efficiency score for aggregating and analyzing data or using cross efficiencies as an analogy for the Desirability Index in MCDM theory and, in this research, are referred to as Classical DEA/MCDM models. The following section outlines the existing literature on using DEA as an MCDM technique.

The Beginning

Stewart [STEW92] provides a survey of MCDM techniques but fails to mention DEA, which prompted Doyle and Green [DOYL93] to address the oversight. Stewart responded in 1994 with a paper titled "Data Envelopment Analysis and Multi Criteria Decision Making" and states:

> *It is, of course, almost inevitable in a review of this nature, that something will be omitted. Actually, had I written the review a year later, I may well have included comment on the relationship between DEA and MCDM thinking, as I have come to share their view that the two fields should be drawn more closely together.* [STEW94]

Until the publication of these papers, both theories had been developed in relative isolation. Stewart's response provided the necessary credibility for DEA to be further used and extended as a technique for MCDM.

Modeling MCDM with DEA

The current body of literature on the topic of using DEA in a decision support setting was investigated and it was found that Cross Efficiencies were an accepted and tested extension to DEA that lends itself well to this type of problem [ORAL91][GREE96][DOYL94]. To be more specific, selection problems or Multi

Criteria Decision Making problems (MCDM) are modeled in DEA by equating a **DMU** to an **Alternative**, **Inputs** to **Criteria to be Minimized** and **Outputs** to **Criteria to be Maximized**. While this approach intuitively makes sense, Doyle notes that the categorization of criteria into Inputs and Outputs is "critical" as the rankings of DMUs change depending on how the model is designed [DOYL93].

A standard DEA run of the data would provide a single efficiency score (sometimes renamed a Desirability Index) that would serve to sort the DMUs into efficient (desirable) and not-efficient (less desirable) units [DOYL93]. Cross-Efficiencies, as will be discussed later, is one method that allows an analyst to rank efficient units.

DEA Term	Equivalent MCDM Term
DMU	Alternative
Input	Criteria to be Minimized (i.e. consumables or negative outcomes from making the decision)
Output	Criteria to be Maximized (i.e. positive outcomes from making the decision)
Efficiency Score	Desirability Index

Table 3 – DEA/MCDM Term Comparison

A number of novel DEA applications have been reported in areas that are not considered traditional efficiency studies. That is to say, the DEA models developed do not attempt to capture efficiencies of production processes, as is traditionally the case with Bank Branch or franchise efficiency studies. Instead, DEA models have been developed that act as strict data analysis tools, where the inputs and outputs of the model are simply relevant variables or criteria used to make decisions or evaluate overall attractiveness of decision alternatives. Such studies include the selection of Mutual Funds [CHEH98], identification of companies close to bankruptcy [SIMA99], selection of appropriate technologies [BAKE97], evaluation of risk tolerance [HOSS04] and the evaluation and selection of industrial R&D projects [ORAL91]. All of these studies have used DEA models and techniques to

simultaneously compare multiple selection criteria (inputs and outputs) for the purpose of creating a shortlist of the most suitable decision alternatives.

2.4.1 What DEA Provides

The output of a DEA analysis provides some interesting information. Each DMU is assigned a relative efficiency score, between zero and one. Only inefficient units will have a score less than one. In effect, the DEA run will separate the DMUs into one of two groups, Efficient (Score = 1.0) and Inefficient (Score < 1.0). This works well in a normal efficiency study but poses some problems when using DEA as a selection tool for MCDM problems in that the basic methodology does not provide any way to rank efficient units, although many extensions have been developed to address this issue (See Section 2.4.6).

Virtual multipliers are also reported for each DMU and indicate the relative weight each input and output received when calculating the overall efficiency score. These multipliers are seldom used in a Classical DEA/MCDM analysis, but they play a central role in this research.

To summarize, a linear program is created for each DMU that is being analysed that attempts to maximize its overall efficiency score (which is calculated as a weighted sum of outputs over inputs) by selecting weights for each output and input such that when those weights are applied to all other DMUs, the resultant efficiency score for those other DMUs is feasible (i.e. between 0 and 1). This is called Unrestricted DEA because there are no restrictions placed on the linear program when choosing weights other than a positivity requirement.

2.4.2 The Self Appraisal and Peer Appraisal Analogy

The technique of using the Classical DEA formulation for modeling MCDM problems is effective, as it can be easily understood by considering the analogy of self-appraisal. DEA will assign an efficiency score to a DMU that makes it look 'as good as possible' when compared to other DMUs. From the perspective of an

individual DMU then, we can say that it is appraising itself [DOYL95].

The drawback of this technique is that there most likely will be numerous efficient DMUs identified from the analysis and each of those DMUs will have an efficiency score of one. This makes it difficult to select among the top results as it leaves room for selection bias and/or requires another justifiable technique for ranking DMUs. To remedy this situation, Doyle suggests the application of Sexton's concept of cross efficiencies [SEXT86]. Cross efficiencies build on the analogy of DMUs appraising or scoring themselves by allowing each DMU to, in effect, score each and every other DMU. This was accomplished by using the weights one particular DMU chooses to make itself look 'as good as possible' in its self-appraisal, on the Inputs/Outputs of every other DMU and recalculating new efficiency scores. This is repeated for all combination of weights and Inputs/Outputs creating an n x n Cross Efficiency Matrix (CEM) containing 'self' and 'peer' appraisals. At this point, the literature proposes that the analyst would average the cross efficiency scores (or take the mean etc. - see [SEXT86]) to calculate one representative cross efficiency score.

By the nature of the formulas involved in calculating cross efficiencies, the individual cross efficiency values are guaranteed to be equal to or lower than the efficiency score and thus provides a convenient way to rank efficient DMUs.

2.4.3 Cross Efficiencies

Cross-efficiency is defined as the efficiency score computed for one particular DMU using the weights chosen by another DMU. A DMU can rate itself highly, that is, it can have a high efficiency score, but it can also be rated quite low by the majority of the other DMUs [SEXT86].

DMU	1	2	3	4	5	6
1	1.000	0.884	0.759	0.649	1.000	1.000
2	0.194	1.000	0.259	0.197	0.414	0.331
3	0.476	0.286	1.000	0.571	0.762	0.490
4	0.250	0.062	0.337	1.000	0.615	0.255
5	0.231	0.026	0.243	0.333	1.000	0.417
6	0.864	0.529	0.579	0.502	1.000	1.000

Table 4 – Sample Cross Efficiency Matrix for Power Plant Problem

Cross-efficiencies are usually presented in the form of an n-by-n cross-efficiency matrix (CEM), where n is the total number of DMUs being evaluated. Table 4, reproduced from [DOYL93] presents the CEM for the Power Plant problem discussed in Section 5.2. The usual efficiency scores (self-rated efficiencies) are located along the main diagonal of the CEM. Further analysis of the CEM shows that by reading across row k, we get a sense of how DMU_k rates every other DMU. Also, reading down column j we can see how DMU_j is rated by each of the other DMUs. In a typical analysis, these cross-efficiency values are averaged along each column and row (excluding the diagonals) to obtain an overall cross-efficiency score.

$$E_{kj} = \frac{\sum_{r=1}^{s} u_{rk} Y_{rj}}{\sum_{i=1}^{m} v_{ik} X_{rj}} \tag{2.3}$$

The cross-efficiency of DMU_j as measured by DMU_k is computed with Equation (2.3) as the ratio of weighted output to weighted input when we use the input and output values of DMU_j and the weights derived for DMU_k. This ratio is simply the classic form of efficiency used in the original CCR model (See Section 3.0). Sexton et. al. also made the observation that the weights chosen for an individual DMU are not unique when an efficiency score of 1.0 is achieved. The arbitrariness of these weights led them to propose a goal programming technique to make the selection of the weights more appropriate.

Aggressive/Benevolent Formulations

The selection of appropriate weights by DEA when assigning efficiency scores can be accomplished by defining two goals: 1) to choose weights that maximize a DMU's efficiency score and 2) to choose weights that make cross-efficiencies as low as possible (also called the *Aggressive* formulation). The secondary goal only comes into play once the primary goal is reached. An analogous *Benevolent* formulation defines the secondary goal as "choosing weights that make cross-efficiencies as high

as possible" [SEXT86]. Both of these formulations are intuitive extensions to the analogy of cross-efficiencies being equivalent to peer-appraisals. Using the Aggressive or Benevolent formulations is equivalent to appraising your peers harshly or forgivingly with resulting scores reflecting that tone.

Other Extensions to Cross-Efficiency Theory

Modified Cross-Efficiency was developed by Hibiki et. al. in 1993 to avoid the goal programming approach of Sexton while still solving the problem of arbitrary weights [HIBI94]. The technique involves calculating minimum and maximum values of cross-efficiency and then explicitly locating the cross-efficiency value within this range. Hibiki also applies modified cross-efficiency results to rank efficient DMUs.

Doyle and Green in 1995 also applied cross-efficiency theory to the problem of improving the discrimination among DMUs [DOYL95a]. Also in 1995, Doyle applied cross-efficiency theory to the problem of Multi Criteria Decision Making (MCDM). His basic premise was to allow regular DEA to act as an idealized process of self-evaluation where each DMU chooses weights to maximize its own desirability, while maintaining feasibility. Cross-efficiencies were then used to simulate idealized peer-evaluation, where each alternative DMU is allowed to rate every other alternative [DOYL95b]. The method was applied to the problem of selecting a University for enrollment based on a number of scored criteria. The analysis proved to be more insightful than a traditional DEA analysis and the results provided a viable screening tool for the problem.

Oral et. al. were the first to apply the concepts of efficiency and cross-efficiency to the problem of selecting R&D projects to comprise an R&D program [ORAL91]. Row and column average scores from the CEM were used in conjunction with self-efficiency scores to pick the top candidates for the program. Candidates were placed into the program until the limited funding was exhausted. Green et. al. extended Oral's work by 'moderating' the cross-efficiency scores. That is to say, the row and column averages were not used. Instead, a weighted average was used where the weights were proportional to a DMUs overall rating. Therefore, a DMU with a

higher overall rating would influence another DMUs' cross-efficiency score more than a DMU with a lower overall rating [GREE96].

Baker and Talluri used cross-efficiency analysis in a MCDM problem for the selection of advanced manufacturing technology [BAKE97]. The model and results were found to be more robust than the original solution in [KHOU95], which only used self-efficiencies.

It should be noted that the selection of cross-efficiency methods will change the calculated cross-efficiency values and in most cases will change the ranking of DMUs. The selection of the appropriate cross-efficiency method is case specific and multiple methods may be appropriate for a particular problem.

2.4.4 Limitations of DEA in MCDM

Unrestricted DEA may assign widely different weights to inputs and outputs for each DMU so as to make the efficiency score of an individual DMU as attractive as possible. No a priori values are required to be assigned to the factor weights.

In real world applications, this tends to generate weights that may be unrealistic and difficult to accept by management. It becomes difficult to explain the widely differing weights on the same factor when assessing different DMUs. Numerous weight restriction methodologies have been developed to address this issue.

2.4.5 Weight Restriction Techniques

A number of methods have been put forward to deal with the problem of unconstrained weights. The cone ratio model was suggested by [CHAR89], which generalized the original CCR model formulation given in [CHAR78]. The cone ratio model requires that input and output weights be restricted within given closed cones. Defining the bounding cones provides for a variety of restrictions to be imposed on factor weights.

The "assurance region" principle proposed by Thompson in [THOM90] restricts factor weights by comparing the weight of one output to the weights of all other

outputs. Input weights are also compared in a similar fashion. Ratios between the various weights are estimated based on observation and/or expert opinion.

Roll et. al [ROLL91] suggest an analysis of the upper and lower bounds obtained from an unconstrained DEA formulation of factor weights. These ‘bounds’ are then injected into a new DEA formulation with additional constraints placed on the multipliers to keep them within the observed upper and lower limits.

All of the presented techniques on factor weight restriction attempt to solve two of the drawbacks of an unrestricted DEA analysis. The first drawback is that unrealistic factor weights can make a DMU appear to be efficient even though the source of efficiency is allocated strictly to a few inputs and outputs (i.e. near zero weights assigned to many other inputs/outputs).

The second drawback is that the application of the Simplex method to solve the resulting linear programs only provides the first optimal and feasible solution. It is well known that a large number of alternative optima may exist (different factor weight vectors) when the number of DMUs is at least twice as large as the number of factors [BANK84].

The problem with using weight restrictions to overcome the two drawbacks mentioned above lies in the fact that there is a great difficulty in transforming expert opinion into linear programming constraints. Setting up ratios constraining factor weights or relative importance relationships among factor weights injects an element of analyst or management bias on the functional form of production space for a particular context. This contradicts one of the major advantages of DEA in that it does not require any a priori knowledge of these relationships.

Even though weight restrictions may seem reasonable, they detract from one of the strongest selling points to DMUs which are being evaluated which is their freedom to choose weights for their inputs and outputs that make them appear in the best possible light [DOYL94]. Also, as in the problems that this research addresses, the analyst does not always have access to expert opinion, which may prevent the application of weight restriction techniques.

2.4.6 Ranking of Efficient Units

There is a growing interest in ranking techniques and methods. This thesis presents a new methodology and models for ranking decision alternatives. The problem to be solved in ranking is to find a set of weights that can be used to rank alternatives, that is also defendable as a better approach than any presented so far. Presented in this section is a review of the current ranking methods as developed in the DEA context.

Adler, Friedman and Sinuany-Stern provide a comprehensive review of ranking methods in the data envelopment analysis context, categorizing the methods into six basic areas: 1) Cross-Efficiency, 2) Super-Efficiency, 3) Benchmarking approaches, 4) Multi-variate statistical approaches, 5) Ranking of inefficient units and lastly 6) Combining DEA and MCDM approaches [ADLE02].

Each of these approaches assumes that we have a finite number of decision alternatives to rank, a finite number of criteria on which to compare and values for each criterion and for each alternative. The first four approaches are described below as they pertain specifically to the topic of this thesis. The fifth area is not covered as it only deals with the problem of ranking inefficient units which presupposes that the standard DEA efficiency score is not a valid method for this purpose and the sixth approach contradicts our basic assumption of not requiring the incorporation of expert opinion into the process as discussed in Section 2.4.

Cross Efficiency theory and application to the problem of ranking was covered in Section 2.4.2 and 2.4.3. The basic premise is to allow each alternative to choose weights that make them as attractive as possible and then to use these weights from each individual alternative and apply them to all other alternatives. This leads to as many efficiency scores for each alternative as there are alternatives, which when averaged, can be used to rank all the alternatives. Cross-efficiencies are an intuitive method for ranking DMUs in that the approach takes into account many perspectives and derives a common set of weights.

Super-efficiency was developed by Andersen and Petersen in 1993 in response to the fact that a traditional DEA analysis cannot distinguish (and thus rank) the efficient

DMUs because they all have a score of 1.0 [ANDE93]. The solution they proposed was to remove the DMU under investigation from the reference set used in calculating the optimal set of weights. In this way, it is possible for a DMU to achieve an efficiency score of greater than 1.0 thus allowing for ranking of efficient units. Of course, inefficient units maintain their ranking.

The Super-Efficiency formulation is identical to the standard CCR formulation except for the removal of the rth constraint as seen in (2.4).

$$\begin{gathered}\max h_o = \sum_{r=1}^{s} u_r y_{ro} \\ s.t. \sum_{i=1}^{m} v_i x_{io} = 1 \\ -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} \le 1; \quad j = 1,\ldots,n,\ j \ne r \\ u_r, v_i \ge 0\end{gathered} \tag{2.4}$$

The Super-efficiency model has been used in a number applications despite the fact that it suffers from several known problems, the main one being that the method uses the DEA efficiency score to rank when this score is calculated for each DMU from individual weights, not one set of consensus weights.

Benchmarking methods are based on the simple concept of ranking efficient DMUs by measuring their importance as a benchmark for inefficient DMUs. There are many ways to do this and no one method makes any more sense than the other. Why should one DMU rank higher than another DMU when they are both deemed to be DEA efficient but one is used as a benchmark more than the other? Based on this argument, a particular DMU benefits based on the simple fact that more inefficient DMUs refer to it. This can be counter-intuitive in certain contexts when it is equally rational to rank a DMU higher based on distinctness (i.e. less inefficient DMUs refer to it).

Khoo and Sowlati present new methods to rank efficient DMUs based on defining new frontiers that envelope the standard DEA empirical frontier. In [KHOO02], a

new model for efficiency evaluation is introduced that enhances DEA by incorporating a stochastic component to derive a new expected frontier. Using this approach Khoo ranks efficient DMUs according to their standard deviation and their importance as reference units. Sowlati takes a different approach. If the inputs and outputs of efficient units can be varied within a specified range, then it is possible to find other combinations of inputs and outputs from which new, “artificial” DMUs can be created. These new DMUs are constrained to be more efficient than the DEA efficient unit from which they were created thus defining a new “Practical Frontier” while utilizing management input. The new frontier, formed mostly from the new, artificial DMUs, thus ranks the efficient units which will now have scores less than 1.0 [SOWL04].

Another attempt at addressing the ranking problem in the context of DEA is to use multi-variate statistical methods on the inputs and outputs used in a DEA analysis to converge on a common set of weights that can then be used for ranking. These methods such as the canonical correlation analysis (used in [FRIE97]), linear discriminant analysis [SINU94] and discriminant analysis of ratios [SINU98] perform statistical regression type calculations on the same inputs and outputs used in the DEA analysis in an attempt to derive a common set of weights.

While each of these methods and techniques provide for ways to rank DMUs under certain contexts and with specific assumptions, no one method can claim to be the complete solution to the question of ranking [ADLE02].

2.4.7 One-Sided DEA Models

There are many examples in the literature of DEA studies and applications that use a one-sided approach. A brief survey is included here but we limit ourselves to those references that attempt to use one-sided DEA as a means of aggregating a number of disparate performance measures. This is outside the usual context of efficiency or productivity analysis where DEA has its roots. The main purpose of these studies is to use DEA to combine several individual performance measures into a single score, where that score is then used in the evaluation of the units being analysed.

Greenberg et. al present a one-sided DEA model in the context of a generalized multiple criteria control model where they aggregate many common ratio measures of performance into a single score [GREE87]. The study is performed with the desire to elicit a subset of performers that are Pareto optimal and using those performers as benchmarks for non-optimal units.

Despic addresses the issue in [DESP02] of ‘strange ratios’ by enumerating all possible ratios and removing any illogical ratios from the analysis, thus creating a one-sided DEA model of ratios called DEA-R.

Thanassoulis et. al, building on the work of Greenberg and Nunamaker [GREE87], compare the assessment of performance using several ratio Performance Indicators (PIs) to a standard DEA approach. The approach used in the creation of PIs is to formulate every possible ratio of single outputs to single inputs thereby creating a combinatorial explosion of PIs from a relatively small number of inputs and outputs [THAN96]. Despic used this same approach to move from a weighted sum of outputs divided by the weighted sum of inputs efficiency measure (two-sided DEA) to the weighted sum of discrete ratios, which can be interpreted as a one-sided model [DESP02].

Jessop analyses the performance of thirty international airlines through the use of a simple weighted sum of attributes where the weights used in the aggregation are found using a linear program inspired by DEA [JESS01]. The resultant model is setup like a one-sided DEA model but instead of determining weights in the usual context of maximizing a summary score, Jessop diverges and determines a set of weights that maximizes the entropy of the distribution of performance scores. The argument made is that calculating weights in this way will minimize the discrimination between airlines and his results verified this assumption.

Hosseini presents a one-sided DEA model to determine a summary risk tolerance measure based on aggregating responses to 24 individual psychological questions. In this application, only 2 of the questionnaire results coding values had to be

inverted to accommodate the one-sided model. The DEA efficiency score was then used to rank the DMUs from most risk tolerant to least risk tolerant [HOSS04].

Rouatt uses a two-stage DEA approach to analyse the efficiency of bank branches where he combined the results of 3 separate DEA models (productivity, profitability, intermediation) using a second stage One-Sided DEA model to derive one summary measure to sort branches. While the results showed only 7% of the branches to be efficient (thus allowing ranking of 93% of the branches to be ranked), the identical score of efficient units still prevented complete ranking [ROUA03].

All of the studies presented in this section utilize a one-sided DEA model, some by chance and others by design – this, in and of itself is not new. Each of these precedents adds credibility to the use of one-sided DEA models for sorting DMUs and can be seen as a valid method for aggregating many disparate measures of performance into one summary measure by means of the DEA mechanism.

The difference between previous work in this area and the models and methods presented in this thesis is the focus and the reason for using a one-sided model. We are concerned with weights – not efficiency scores per se. The reason for this is two-fold, 1) to allow for comparison to expert weights and 2) to allow for 'stacking' of weights for consensus. Each of the presented works from this section is a prime candidate for applying the models and methods developed in this thesis.

2.5 Summary of Literature Reviewed

This section has been focused on providing a review of the relevant literature as it pertains to the broad subject area of Ranking. We started by discussing the general concepts of decision making and identifying the three types of decisions: choosing, sorting and ranking. We surveyed the areas from the MCDM literature that are relevant to this work and found that at the highest level two main ideas are relevant: 1) that human preference can be coded into the weight importance of criteria values and 2) the results from 1) can be aggregated and interpreted as group preferences. Special attention and focus was paid to the areas from Data Envelopment Analysis

theory that are being extended and exploited in this area and for this work. The main property being the ability to calculate ‘best weights’ for individual performers.

Data Envelopment Analysis on its own is a broad discipline which has its roots in Efficiency/Productivity analysis. The next Section surveys the area from this perspective and should be understood for completeness. If the reader wishes, they can move ahead to Section 4.0 which presents the OSD models and RC methodology which, after standardizing the structure of such problems, shows how to combine the ‘best weights’ property of DEA with the two main ideas from MCDM to create a new way to rank by consensus.

3.0 DATA ENVELOPMENT ANALYSIS

"I am so efficient – I'm ficient"

3.1 ORIGINS OF DEA

Charnes, Cooper and Rhodes [CHAR78] introduced the first DEA model, known as the CCR model named after its authors, in 1978. However, much credit is due to Farrell's 1957 publication of an optimization method of mathematical programming using a single-input single-output technical efficiency measure [FARR57]. The CCR model was developed to measure the relative efficiency of a set of Decision Making Units (DMUs) with an identical set (although different values) of input and output variables. The DEA technique is especially useful in cases of multiple inputs and/or outputs.

Since the introduction of DEA, many developments, extensions and novel applications have been published. Today, there are over 3,000 articles and several books available on the subject [COOP00], the most comprehensive text on the subject was released in 2000 by Cooper, Seiford and Tone titled "Data Envelopment Analysis – A Comprehensive Text with Models, Applications, References and DEA-Solver Software".

The DEA methodology and models have become essential tools for any Operations Research professional. Many undergraduate and graduate curriculums in Engineering and Business Management now include DEA and new applications and extensions continue to be researched and developed.

Numerous software packages have been developed to aid with the analysis of DEA problems. Part of this research was conducted using DEA Solver 1.1 and validated using ProDEA 1.0. The remainder was performed using the LINDO software package, Version 6.

3.2 RATIO DEFINITION OF EFFICIENCY

The CCR model is based on the classic engineering definition of efficiency. As quoted from [CHAR78], efficiency is defined as 'the ratio of the actual amount of heat liberated in a given device to the theoretical maximum amount which could be liberated by the fuel'. In symbols, E_e=Y/MPY, where E_e = Efficiency score (the 'e' subscript indicates Engineering definition), Y = Actual Output, and MPY = Maximum Possible Output. This definition makes sense and is applied in the field of combustion engineering and other processes where the theoretical maximum is known, for example, an auto plant can make at most X units per day. In these examples, we have specific engineering data and mathematical models for determining the MPY, but this definition tends to break down when trying to measure the efficiency of DMUs because it is difficult (some say impossible) to put a number on the MPY when people are involved in the production process.

The difficulty in assigning a value to MPY for measuring the efficiency of DMUs lead Charnes et.al to consider an alternate definition of efficiency, taken from Systems Theory, which defines it as, E_s = Output/Input (where the 's' subscript of E indicates Systems Theory definition).

$$0 \le Efficiency = \frac{Virtual\,Output}{Virtual\,Input} \le 1 \tag{3.1}$$

Charnes et.al make their case for the CCR model based on three compelling arguments [CHAR78]:

1. E_s definition of efficiency is valid under certain contexts
2. $0 \le E_s \le 1$ and $0 \le E_e \le 1$
3. In the special case of 2 DMUs, 1 Output and 1 Input (identical for both DMUs), the CCR formulation of efficiency reduces to the E_e definition

The Output/Input definition of efficiency allowed for the creation of the original CCR model, described next, which has successfully been applied in many diverse

application areas, leading to new insight and understanding of complex production environments, enabling benchmarking and practical advice for improving the future performance of DMUs.

3.2.1 Types of Efficiency

Price efficiency is the efficiency of an organization to purchase the inputs that meet desired quality standards at the lowest price. *Allocative efficiency* gives a measure of whether an organization is using the optimal mix of inputs to produce the optimal mix of desired outputs (for example a bank's use of automatic teller machines versus reliance on tellers or customer service representatives). *Technical efficiency* refers to the efficiency or a DMU in converting inputs into outputs. Technical inefficiency exists when it is possible to produce more outputs with the same inputs used or to produce the present level of outputs with fewer inputs. *Scale efficiency* examines whether an organization is operating at its optimal scale size. Producing more or fewer goods or services than the optimal level results in added costs only due to the volume and size. A comprehensive efficiency analysis approach requires explicitly recognizing, analyzing and managing some or all of these components.

3.3 THE CCR MODEL

The development of the DEA methodology stems from the usual measure of productivity, which involves the computation of the ratio of outputs to inputs. The formulation of a relative efficiency measure, or the ratio of weighted outputs to weighted inputs, was introduced to account for the existence of multiple inputs and multiple outputs. Charnes, Cooper and Rhodes [CHAR78] recognized the difficulty in determining a common set of weights to measure the performance of such a unit. The common set of weights proved to be a problem since one unit's inputs and outputs are not likely to be valued in the same manner as those of another unit. They proposed that each unit should choose weights that allows it to be shown in the most favourable light as compared to all other units. This led to the following formulation:

3.3.1 CCR Formulation

The resultant fractional program expressed in (3.2) extends the single input to single output efficiency measure to the case of multiple inputs and outputs by introducing weights (u_r, v_i) that are calculated as opposed to determined a priori.

$$\max h_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

$$s.t. \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1; \quad j = 1, \ldots, n, \tag{3.2}$$

$$u_r, v_i > 0; \; r = 1, \ldots, s; \; i = 1, \ldots, m.$$

where
u_r = weight given to output r
y_{ro} = amount of output r from DMU_o
y_{rj} = amount of output r from DMUj
v_i = weight given to input i
x_{io} = amount of input i from DMU_o
x_{ij} = amount of input i from DMUj

Consider the general case where *n* DMUs: (DMU_1,$DMU_{2,\ldots,}$$DMU_{n)}$ are to be relatively evaluated based on *m* Inputs (X_1, $X_{2,\ldots,}$ X_m) and *s* Outputs (Y_1, $Y_{2,\ldots,}Y_s$). Solving the above fractional program for each of the *n* DMUs will provide a single relative efficiency score for each. For the single input, single output case, we can graphically represent the solution as depicted in Figure 1.

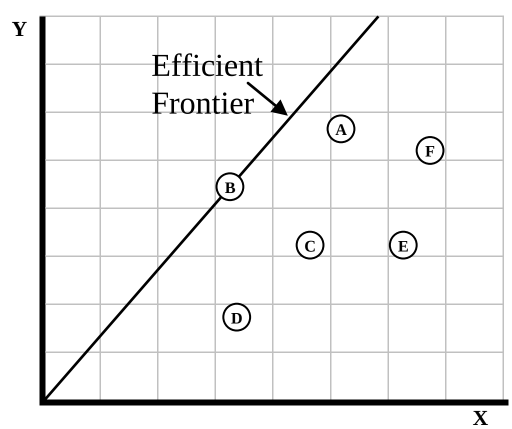


Figure 1 – CCR Illustration

The above fractional program formalizes the CCR definition of efficiency as the maximum value of a weighted sum of outputs divided by a weighted sum of inputs given that the same calculation for all DMUs leads to a score between zero and one.

To solve the algebraic model it is first necessary to convert the model into linear form so that the methods of linear programming can be applied. This can be achieved by setting the denominator in the objective function to one and maximizing the numerator. The resulting linear program (LP) (3.3) can easily be solved using standard LP software.

$$\begin{aligned} \max h_o &= \sum_{r=1}^{s} u_r y_{ro} \\ s.t. \sum_{i=1}^{m} v_i x_{io} &= 1 \\ -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} &\le 0; \quad j = 1,\ldots,n \\ u_r, v_i &\ge 0 \end{aligned} \tag{3.3}$$

Equation (3.3) presents the traditional *Multiplier* form of the CCR model. The solution to this linear program is identical to the solution from the original fractional program (efficiency scores and weights) however now that it is in the standard form, we are free to use the properties of linear programming to derive the dual formulation, as expressed in (3.4).

The dual, also known as the *Envelopment* form, provides for a different interpretation of the problem and returns the representative hyperplanes that define the efficient frontier. In the two dimensional example from Figure 1, the hyperplane appears as a line but the envelopment property is clearly visible. It is this envelopment property that distinguishes DEA from other regression based measures and sets it apart in identifying best performers.

$$
\begin{aligned}
&\min \quad \theta \\
&s.t. \quad \theta \cdot x_{io} - \sum_{j=1}^{n} \lambda_j . x_{ij} \geq 0 \qquad i = 1,\dots,m \\
&\qquad \sum_{j=1}^{n} \lambda_j \cdot y_{rj} \geq y_{ro} \qquad r = 1,\dots,s \\
&\qquad \lambda_j \geq 0 \qquad j = 1,\dots,n,\ r = 1,\dots,s,\ i = 1,\dots,m
\end{aligned} \tag{3.4}
$$

Transformation of the dual problem into standard linear programming form requires the addition of slack variables s^- and s^+. Slack variables are needed to convert inequality constraints from the primal to equality constraints in the dual. Slacks have an intuitive meaning in DEA and correspond to specific input/output efficiencies. The standard linear program for the dual CCR Input oriented model with slacks is presented in (3.5).

$$
\begin{aligned}
&\min \quad \theta \\
&s.t. \quad \theta \cdot x_{io} - \sum_{j=1}^{n} \lambda_j \cdot x_{ij} - s_i^- = 0 \qquad i = 1,\dots,m \\
&\qquad \sum_{j=1}^{n} \lambda_j \cdot y_{rj} - s_r^+ = y_{ro} \qquad r = 1,\dots,s \\
&\qquad \lambda_j, s_r^+, s_i^- \geq 0 \qquad j = 1,\dots,n,\ r = 1,\dots,s,\ i = 1,\dots,m
\end{aligned} \tag{3.5}
$$

Definition of CCR Efficiency: A DMU is said to be CCR Efficient if and only if $\theta^*=1$ and all slacks are 0, otherwise, the DMU is deemed to be CCR Inefficient.

Charnes, Cooper and Rhodes [Char78] made the results more precise through a two-phase formulation that maximized the slack variables without affecting the minimization of θ. This resulted in the following objective function:

$$
\begin{aligned}
&\min \quad \theta - \varepsilon \sum_{i=1}^{m} s_i^- - \varepsilon \sum_{r=1}^{s} s_r^+ \\
&s.t. \quad \theta \cdot x_{io} - \sum_{j=1}^{n} \lambda_j \cdot x_{ij} - s_i^- = 0 \qquad i = 1,\dots,m \\
&\qquad \sum_{j=1}^{n} \lambda_j \cdot y_{rj} - s_r^+ = y_{ro} \qquad r = 1,\dots,s \\
&\qquad \lambda_j, s_r^+, s_i^- \geq 0 \qquad j = 1,\dots,n,\ r = 1,\dots,s,\ i = 1,\dots,m
\end{aligned} \tag{3.6}
$$

The first Phase of the solution involves the standard minimization of θ to find θ^*. By the duality theorem of linear programming, we know that θ^* is equal to the optimal objective value h_o. The second Phase then proceeds to use this optimal value of θ^*, fixing its value while maximizing the sum of input excesses and output shortfalls (s^- and s^+). Note: Equation (3.6) introduces the concept of a non-Archimedian constant (ε) needed to allow the solution by computer where ε is defined as a number for which there exists no real positive number between it and zero [COOP96].

The CCR measure of efficiency assumes a Constant Returns to Scale (CRS) relationship between Inputs and Outputs. This implies that if (x,y) is a possible combination of input and outputs, then (kx, ky) are also possible, where k is a positive scalar. This assumption is used in the calculation of θ^* which determines if a DMU is efficient or inefficient. The additional benefit of DEA is that it also provides direction for inefficient DMUs on how to become efficient (i.e. move to the frontier). This can be accomplished by either a proportional reduction of inputs (keeping outputs fixed) or by a proportional increase in Outputs (keeping inputs fixed). The former is defined as an Input Oriented CCR model and the latter an Output Oriented CCR model.

3.3.2 CCR Input Oriented Model

The examples and derivations up to this point have been for an Input Oriented CCR model. This model is pictured in Figure 2 illustrates the projection of inefficient DMUs onto the efficient frontier. The CCR model assumes a radial projection mechanism, which implies that a constant and proportional contraction of all inputs, relative to θ^*, when applied to an inefficient DMU will make it efficient.

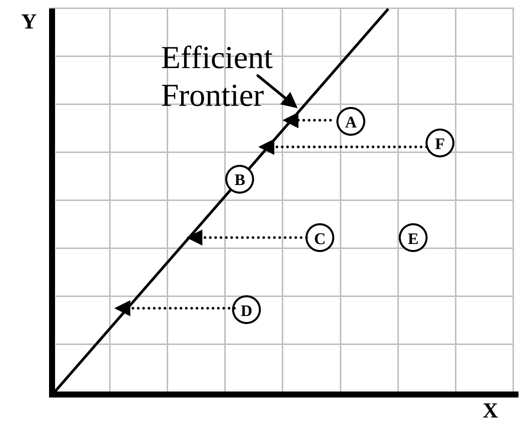


Figure 2 – CCR Input Oriented Projection

3.3.3 CCR Output Oriented Model

Using the same graphical example for Input Oriented model, we can derive the analogous Output Oriented CCR model. In this case, the θ^* remains the same, however inefficient DMUs are now projected to the efficient frontier through a proportional augmentation of outputs, while keeping inputs fixed.

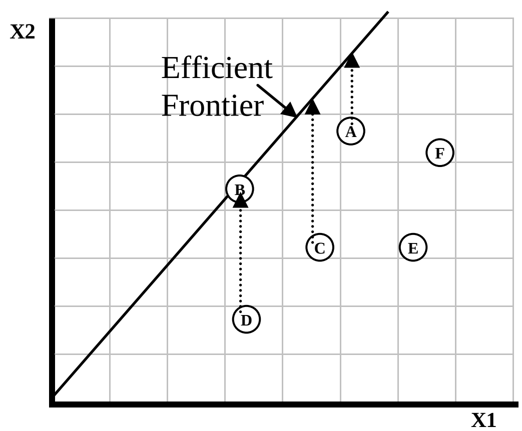


Figure 3 – CCR Output Oriented Projection

The primal (multiplier) form of CCR output oriented is as follow:

$$
\begin{aligned}
&\min \quad q_o = \sum_{i=1}^{m} v_i . x_{io} \\
&s.t. \quad \sum_{r=1}^{m} u_r . y_{ro} = 1 \\
&\sum_{i=1}^{m} v_i . x_{ij} - \sum_{r=1}^{s} u_r . y_{rj} \geq 0 \qquad j = 1, \ldots, n \\
&u_i, v_r \geq \varepsilon \qquad i = 1, \ldots, m, \;\; r = 1, \ldots, s
\end{aligned}
\tag{3.7}
$$

and the dual for it is formulated as:

$$
\begin{aligned}
&\max \quad z_o = \phi + \varepsilon \cdot \sum_{r=1}^{s} s_r^{+} + \varepsilon \cdot \sum_{i=1}^{m} s_i^{-} \\
&s.t. \qquad \phi \cdot y_{ro} - \sum_{j=1}^{n} \lambda_j \cdot y_{rj} + s_r^{+} = 0 \qquad r = 1, \ldots, s \\
&\sum_{j=1}^{n} \lambda_j \cdot x_{ij} + s_i^{-} = x_{io} \quad i = 1, \ldots, m \\
&\lambda_j, s_i^{-}, s_r^{+} \geq 0 \qquad j = 1, \ldots, n, i = 1, \ldots, m, r = 1, \ldots, s
\end{aligned}
\tag{3.8}
$$

From the dual model, we see that output augmentation is accomplished through the variable ϕ. If ϕ is greater than 1.0 and/or the slacks are not zero, then the DMU under investigation is inefficient. To improve and shift the DMU towards or onto the frontier, a proportional increase of ϕ for all outputs is required, followed, potentially, by an adjustment of individual slacks.

A DMU is characterized as efficient in an input oriented CCR model, if and only if it is characterized efficient in the corresponding output oriented CCR model.

To complete a comprehensive review of DEA literature, it is important to touch on the various extensions and enhancements to the basic theory that have been developed and applied over DEA's 26-year history. DEA's broad application scope extends beyond the Selection Problem setting of this thesis which has required the development of many new DEA models, most notably a Variable Returns to Scale model (the BCC model named after its authors) [BANK84], and Additive and Multiplicative models to name a few. Extensions to handle non-discretionary and categorical variables as well as different orientations, Input vs. Output, have been

published and successfully applied. For an in depth analysis and sample applications of DEA in an efficiency/productivity setting, please refer to [COOP00].

3.4 OTHER DEA MODELS

This section provides an overview of three other common DEA models, building on the core concept embodied in the CCR formulation. The first extension to the model is a Variable Returns to Scale (VRS) model, called the BCC model after its authors which relaxes the CRS assumption of the original CCR model. The remaining two models are the Additive and Multiplicative models, each with its own assumptions, features and benefits.

3.4.1 BCC Model

Banker, Charnes and Cooper developed the BCC model in 1984 (BANK84) to model and estimate a Variable Returns to Scale (VRS) production set. The original CCR formulation evaluates both technical and scale efficiency, combining both measures in a single efficiency score. The BCC model evaluates pure technical efficiency of decision making units and identifies whether a DMU is operating in increasing, decreasing or constant returns to scale. By allowing DMUs operating at different scale sizes to be included in the same efficiency analysis, Banker et. al opened the door for new applications and extensions to DEA.

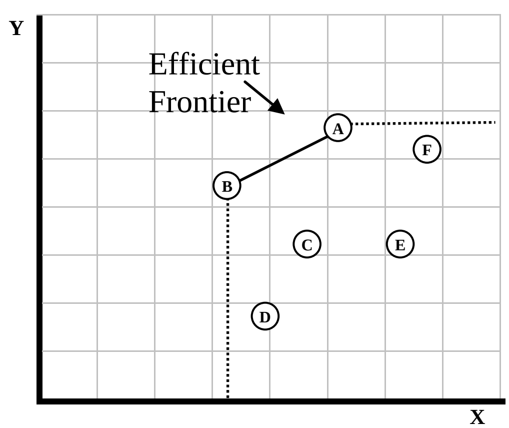


Figure 4 – BCC Illustration

Figure 4 illustrates the VRS envelopment surface for the single input, single output case. Due to the relaxation of the CRS constraint, one additional DMU is found to be efficient in this case as compared to the CCR example. To be more precise, the number of efficient units in a BCC model will always be greater than or equal to the number of efficient units in an equivalent CCR model. In this example, Units A and B are efficient and Units C, D, E, and F are inefficient.

As in the CCR model, the BCC model can also be used in an Input or an Output orientation.

BCC Input Orientation

Equation (3.9) and Figure 5 present the formulation and two dimensional illustration of an input oriented BCC model.

$$
\begin{aligned}
\max \quad & h_o = \sum_{r=1}^{s} u_r . y_{ro} + u_o \\
s.t. \quad & \sum_{i=1}^{m} v_i . x_{io} = 1 \\
& \sum_{r=1}^{s} u_r . y_{rj} - \sum_{i=1}^{m} v_i . x_{ij} + u_o \le 0 \qquad j = 1, \ldots, n \\
& u_r, v_i \ge \varepsilon \qquad r = 1, \ldots, s \quad i = 1, \ldots, m \\
& u_o \quad free
\end{aligned} \tag{3.9}
$$

A DMU is BCC efficient if and only if the efficiency score is 1.0 and all slacks are zero as in the CCR model. It is the presence of the u_o variable in the primal and corresponding constraint ($\sum \lambda_j = 1$) in the dual which distinguishes the VRS from CRS model.

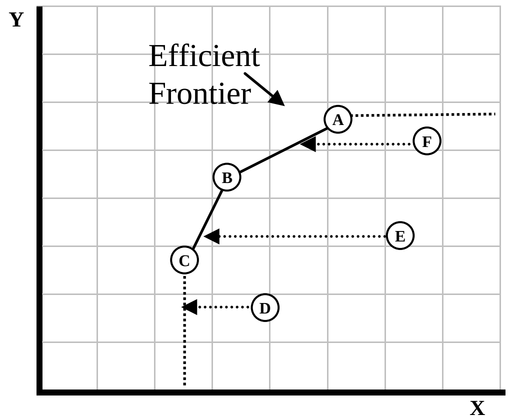


Figure 5 – BCC Input Oriented Projection

The dual formation is defined as:

$$
\begin{aligned}
\min \quad & \theta - \varepsilon \sum_{i=1}^{m} s_i^- - \varepsilon \sum_{r=1}^{s} s_r^+ \\
s.t. \quad & \theta \cdot x_{io} - \sum_{j=1}^{n} \lambda_j \cdot x_{ij} - s_i^- = 0 \qquad i = 1,\ldots,m \\
& \sum_{j=1}^{n} \lambda_j \cdot y_{rj} - s_r^+ = y_{ro} \qquad r = 1,\ldots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \qquad j = 1,\ldots,n \\
& \lambda_j, s_i^-, s_r^+ \geq 0 \qquad j = 1,\ldots,n,\ i = 1,\ldots,m,\ r = 1,\ldots,s
\end{aligned}
\tag{3.10}
$$

BCC Output Orientation

As in the CCR model, the resulting efficiency score for both the Input and Output oriented BCC models provide the same optimal outcome. The difference between the two arises from the desire to either augment output production while maintaining current inputs or the opposite, maintaining output production while consuming less inputs. Graphically, this property can be seen by comparing the projection of inefficient DMUs (C, D, E, F) in Figure 5 and Figure 6 respectively.

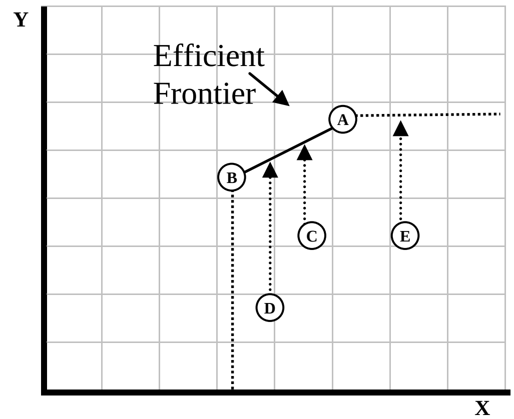


Figure 6 – BCC Output Oriented Projection

Presented below is the primal form of the BCC Output formulation.

$$
\begin{aligned}
\min \quad & q_o = \sum_{i=1}^{m} v_i . x_{io} + v_o \\
s.t. \quad & \sum_{r=1}^{m} u_r . y_{ro} = 1 \\
& \sum_{i=1}^{m} v_i . x_{ij} - \sum_{r=1}^{s} u_r . y_{rj} + v_o \geq 0 \qquad j = 1,\dots,n \\
& u_i , v_r \geq \varepsilon \qquad i = 1,\dots,m, \; r = 1,\dots,s \\
& v_o \qquad \mathit{free}
\end{aligned} \tag{3.11}
$$

The corresponding dual formation is therefore:

$$
\begin{aligned}
\max \quad & z_o = \phi + \varepsilon \cdot \sum_{r=1}^{s} s_r^{+} + \varepsilon \cdot \sum_{i=1}^{m} s_i^{-} \\
s.t. \quad & \phi \cdot y_{ro} - \sum_{j=1}^{n} \lambda_j \cdot y_{rj} + s_r^{+} = 0 \qquad r = 1,\dots,s \\
& \sum_{j=1}^{n} \lambda_j \cdot x_{io} + s_i^{-} = x_{io} \qquad i = 1,\dots,m \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j , s_i^{-} , s_r^{+} \geq 0 \qquad j = 1,\dots,n, i = 1,\dots,m, r = 1,\dots,s
\end{aligned} \tag{3.12}
$$

Analogous to the CCR formulation, a DMU is deemed to be efficient in a BCC setting if and only if $\phi *$ equals one and all slacks are zero.

3.4.2 Additive Model

The Additive model has the same production possibility set as the BCC and CCR models but treats the slacks (input excesses and output shortfalls) directly in the objective function. This shift allows us to move away from the distinction between input and output orientation while trading off the radial efficiency measure found in the CCR and BCC formulations. In an Additive model, the same efficient frontier is found as in the BCC model, however an inefficient DMU is no longer projected to the frontier through a proportional reduction of inputs or augmentation of outputs. Instead, the model projects inefficient DMUs to the frontier through simultaneous adjustments of slack variables, resulting in a step wise movement to the most distant location to the frontier. The main benefit of this approach is the resultant *translation invariance* property of the Additive model which is discussed in Section 3.5.3.

The primal form of the Additive model is:

$$\begin{aligned}
\max \quad & z_o = -\varepsilon \sum_{i=1}^{m} s_i^- - \varepsilon \sum_{r=1}^{s} s_r^+ \\
s.t. \quad & x_{io} - \sum_{j=1}^{n} \lambda_j . x_{ij} - s_i^- = 0 \qquad i = 1,\ldots,m \\
& \sum_{j=1}^{n} \lambda_j . y_{rj} - s_r^+ = y_{ro} \qquad r = 1,\ldots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \qquad j = 1,\ldots,n \\
& \lambda_j, s_i^-, s_r^+ \geq 0 \qquad j = 1,\ldots,n, i = 1,\ldots,m, r = 1,\ldots,s
\end{aligned} \tag{3.13}$$

And its dual:

$$\begin{aligned}
\min \quad & w_o = \sum_{r=1}^{s} u_r . y_{ro} - \sum_{i=1}^{m} v_i . x_{io} + u_o \\
& \sum_{r=1}^{s} u_r . y_{rj} - \sum_{i=1}^{m} v_i . x_{ij} + u_o \leq 0 \qquad j = 1,\ldots,n \\
& u_r, v_i \geq 1 \qquad r = 1,\ldots,s \quad i = 1,\ldots,m \\
& u_o \quad \text{free}
\end{aligned} \tag{3.14}$$

Figure 7 illustrates the efficient frontier and projection of DMU 'C' onto the frontier. The Additive model shares the same variable returns to scale frontier as the BCC model however the resultant projected target differs from either the BCC or CCR models.

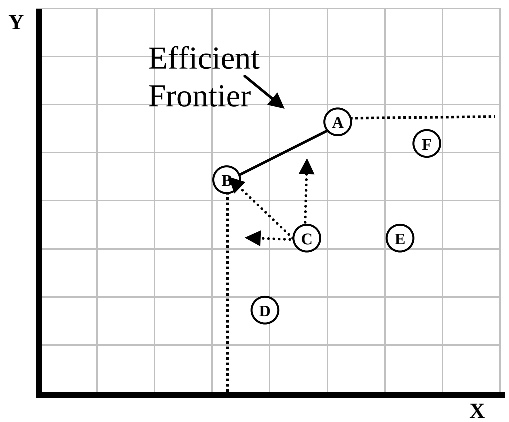


Figure 7 – Additive Model Projection of Unit C

3.4.3 Multiplicative Model

Developed in 1982 by Charnes, Cooper and Seiford, the Multiplicative model is a DEA model in which a multiplicative combination instead of an additive combination of inputs and outputs are used to achieve virtual inputs and outputs. It has a piecewise log-linear envelopment surface [CHAR82].

$$\max \quad \frac{\prod_{r=1}^{s} y_{ro}^{u_r}}{\prod_{i=1}^{m} x_{io}^{v_i}}$$

$$s.t. \quad \frac{\prod_{r=1}^{s} y_{rj}^{u_r}}{\prod_{i=1}^{m} xij^{vi}} \le 1 \qquad j = 1,\ldots,n \tag{3.15}$$

$$u_r, v_i \ge 1 \qquad r = 1,\ldots,s,\ i = 1,\ldots,m$$

By taking logarithms (to any base), the above formulation can be written as a linear program:

$$
\begin{aligned}
&\max \quad && \sum_{r=1}^{s} u_r.\log y_{ro} - \sum_{i=1}^{m} v_i.\log x_{io} \\
&s.t. \quad && \sum_{r=1}^{s} u_r.\log y_{rj} - \sum_{i=1}^{m} v_i.\log x_{ij} \le 0 \qquad j = 1,\ldots,n \\
&u_r, v_i \ge 1 \quad && r = 1,\ldots,s,\ i = 1,\ldots,m
\end{aligned} \tag{3.16}
$$

And the corresponding dual program is:

$$
\begin{aligned}
&\min \quad && -\sum_{i=1}^{m} s_i^- - \sum_{r=1}^{s} s_r^+ \\
&s.t. \quad && \sum_{j=1}^{n} \log x_{ij}.\lambda_j + s_i^- = \log x_{io} \quad i = 1,\ldots,m \\
& && \sum_{j=1}^{n} \log y_{rj}.\lambda_j - s_r^+ = \log y_{ro} \quad r = 1,\ldots,s \\
& && s_i^-, s_r^+, \lambda \ge 0 \qquad i = 1,\ldots,m, \quad r = 1,\ldots,s, \quad j = 1,\ldots,n
\end{aligned} \tag{3.17}
$$

As in other models, a DMU is found to be efficient in the Multiplicative model if and only if its efficiency score is one and all slacks are found to be zero. The resultant efficient frontier takes on a log-linear shape as opposed to the piece-wise linear frontier of other models.

3.4.4 Slacks-Based Measure (SBM) Model

The Slacks-Based Model was introduced by Tone in [TONE01] as a means of dealing directly with the input excesses and output shortfalls of a DMU.

$$
\begin{aligned}
&\min \rho = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} s_i^- / x_{io}}{1 - \frac{1}{s}\sum_{r=1}^{s} s_i^+ / y_{io}} \\
&st \qquad x_o = X\lambda + s^- \\
&\qquad\quad\; y_o = Y\lambda - s^+ \\
&\qquad\quad\; \lambda \ge 0, s^- \ge 0, s^+ \ge 0
\end{aligned} \tag{3.18}
$$

Equation (3.18) presents the fractional linear program for the SBM model. [TONE01] presents how to convert the fractional program from (3.18) into a

nonlinear programming problem and then from there, transforms it into a linear program for the purposes of being able to compare the SBM measure of efficiency to other measures of efficiency.

3.5 DEA EXTENSIONS

The four standard DEA models presented in this section ($CCR_{I\&O}$ & $BCC_{I\&O}$) allow for the modeling of a wide variety of production processes for the purpose of evaluating efficiency and providing real interpretable goals for inefficient DMUs. As is often the case, any particular DEA study will pose its own challenges in modeling a production process, whether that be determining appropriate inputs and outputs, deciding on a model orientation (input or output) and interpreting the results for management. The varied cross section of applications that have been successfully modeled using DEA lends credibility to its effectiveness and robustness as a management tool. This section continues by highlighting three other extensions to the basic DEA theory that have been developed to address more specific requirements that arise in efficiency studies. Their existence and relatively simple mapping to common management issues and production scenarios are further testament to DEA's sound applicability and importance in efficiency analysis.

3.5.1 Categorical Variables

The first extension arises from the fact that in many real world situations, DMUs will be required to be separated into different categories to allow for a fair and accurate evaluation of relative efficiency. For example, if performing a study of bank branches, one variable in the study could be *HasATM* with values being either *Yes* or *No.* Banker and Morey [BANK86b] present a methodology for handling these types of variables using a mixed-integer LP model. The main idea behind these types of models is to evaluate the efficiency of each DMU with respect to the envelopment surface forming from its category and all 'disadvantaged categories', i.e. those operating under the same or worse environment [CHAR97].

3.5.2 Exogenous/Non-Discretionary Variables

Another important extension of DEA is the incorporation of non-discretionary or exogenous variables that were first introduced by Banker and Morey in 1986 [BANK86a]. Exogenous variables are defined as those variables, whether inputs or outputs, that are beyond the control of management. In a typical efficiency analysis, these variables could be values such as store location, weather etc. and while useful and informative to include in an analysis, necessitate a different mechanism for accounting for them if the results of the analysis are to be accepted by the managers of the DMUs being evaluated.

Equation (3.19) presents the linear program for a CCR model incorporating non-discretionary variables where I_D represents discretionary variables. The calculation of θ in this formulation is related directly to the discretionary variables however non-discretionary variables only have an indirect effect on the efficiency score. The same approach can be applied to other DEA models as Charnes et. al details for the BCC and Additive models [CHAR97].

$$
\begin{aligned}
\min \quad & \theta - \varepsilon \sum_{i \in I_D} s_i^- - \varepsilon \sum_{r \in O_D} s_r^+ \\
s.t. \quad & \theta \cdot x_{io} - \sum_{j=1}^{n} \lambda_j \cdot x_{ij} - s_i^- = 0 \qquad i \in I_D \\
& x_{io} - \sum_{j=1}^{n} \lambda_j \cdot x_{ij} - s_i^- = 0 \qquad i \notin I_D \\
& \sum_{j=1}^{n} \lambda_j \cdot y_{rj} - s_r^+ = y_{ro} \qquad r = 1,\ldots,s \\
& s_i^-, s_r^+, \lambda \geq 0 \qquad i = 1,\ldots,m, \quad r = 1,\ldots,s, \quad j = 1,\ldots,n
\end{aligned}
\tag{3.19}
$$

3.5.3 Translation Invariance

The relative property of DEA allows for certain models to demonstrate a translation invariance property that may be desirable under certain contexts. In the case where DMUs are being evaluated and some or all of the inputs and or outputs have negative values (i.e. profit/loss data for bank branches), a BCC or Additive model can be used

to accommodate this type of data. The BCC Input oriented model is translation invariant for its outputs, and correspondingly an Output oriented BCC model is translation invariant in its inputs. This can be clearly seen by inspecting Figure 8 where we can see that a shift up in the whole production space (output dimension) will have no impact on either the efficiency score calculation (based on its relative derivation) and or inefficient DMU projections.

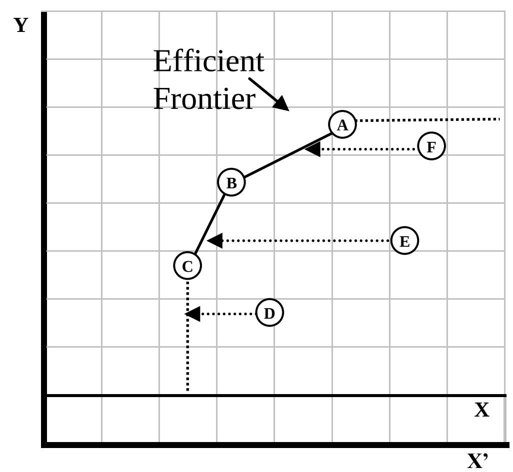


Figure 8 – BCC Input Translation Invariance

Similarly, for the Additive model, relative efficiency scores will not be affected by either a lateral shift of inputs or outputs. Projection of inefficient units in the Additive model is based on movements along slack variables and is not tied to the absolute coordinates of inputs and outputs.

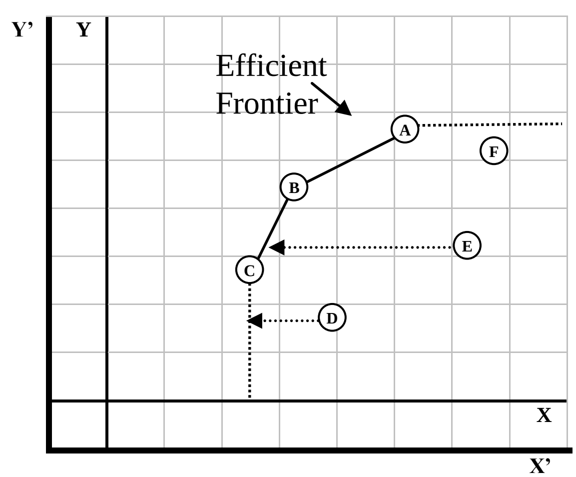


Figure 9 – Additive Model Translation Invariance

3.5.4 Returns to Scale

The CCR model is based on the Constant Returns to Scale (CRS) assumption which states that if activity (x,y) is feasible, then for every positive scalar *t*, (*t*x, *t*y) is also feasible. This assumption is relaxed in the BCC formulation from Section 3.4.1 allowing for DMUs to operate and be evaluated in a Variable Returns to Scale (VRS) setting.

By removing the CRS assumption, DMUs can now demonstrate three different types of scale properties. They can operate in an increasing returns to scale, decreasing returns to scale, or constant returns to scale environment.

3.6 SUMMARY OF DEA REVIEW

In this Section we reviewed the history and major developments of Data Envelopment Analysis in the context of evaluating the efficiency of production units. We presented the formulations and illustrated the mechanics of the mathematics involved for the standard DEA models and their extensions.

The interested reader is directed to three recent works, which provide a complete survey of the subject area:

1. Epistemology of data envelopment analysis and comparison with other fields of OR/MS for relevance to applications [GATT04A].

2. A taxonomy for data envelopment analysis [GATT04B].

3. Data envelopment analysis literature: a bibliography update (1951-2001) [GATT04C].

Furthermore, by far the best textbook on DEA is by Cooper, Seiford and Tone [COOP00] which exposes all aspect of DEA to the reader.

4.0 RANKING BY CONSENSUS

"To me, consensus seems to be the process of abandoning all beliefs, principles, values and policies. So it is something in which no one believes and to which no one objects; the process of avoiding the very issues that have to be solved, merely because you cannot get agreement on the way ahead. What great cause would have been fought and won under the banner, 'I stand for consensus'" – Margaret Thatcher

"Ranking by Consensus" (RC) refers to the concept of rank ordering alternatives based on the consensus opinion of the very alternatives being ranked. While this may seem like circular reasoning, what we are actually doing is determining a set of weights that corresponds to the consensus opinion of the alternatives being evaluated. These weights are then used to score and rank. Therefore, at its root, the Ranking by Consensus Methodology (RCM) is built on the determination of *weights*.

As presented and discussed in Section 2.0 and Section 3.0, DEA and MCDM are also concerned with determining weights. In a typical MCDM problem, the main objective of a solution technique involves the calculation or elicitation of weights, by or from experts that become the basis against which decision alternatives are evaluated. In a typical DEA model, the calculation of these same weights is required, however expert opinion is not necessary for the process.

In DEA, these weights are calculated for each alternative so as to maximize its relative score. The fact that experts' opinion is not required a priori is one of the principal advantages of using DEA in the analysis of MCDM problems. DEA requires no external expert weight preference data although it can be incorporated if available or required (see Section 2.4).

So what is the difference between the weights an expert assigns to each criteria using MCDM theory vs. the weights an individual DMU assigns to its own criteria using a DEA approach? That was the question that provoked the creation of a new method and new models for using DEA with MCDM problems, and a new interpretation of the results. This new approach simplifies the concept of cross efficiencies and puts them into a form that can shed new light on the decision making process, aid in the

selection of weights in a classic MCDM problem and narrow the gap between MCDM assigned weights and DEA derived weights.

The problem with the standard way to model MCDM problems using DEA is that the information one gets from the analysis is the efficiency score and/or cross efficiency scores. For our research the weights derived using standard DEA approaches are problematic (due to DEA's ratio formulation) and we are concerned with weights. Doyle attempted to derive meaning out of the weights output from the standard DEA model by inferring that they are the preferences of the average decision maker. The argument is a good one and gains additional credibility by the fact that the average of those weights applied to each DMU's criteria is equivalent to the average cross efficiency score for a particular DMU [DOYL95].

However, we would propose a new analogy for investigating the weights output from a DEA analysis - an analogy that helps to bridge the gap between MCDM theory and DEA theory. That analogy builds on the concept of self and peer appraisal, to use DEA as a means of evaluating alternatives *in the best possible light* (while maintaining feasibility) but extends the concept through the creation of new methods and models, presented next.

4.1 OBJECTIVES OF RESEARCH

4.1.1 Create a New Methodology for the Ranking of DMUs from a large group

The objective of this research was to investigate the feasibility and accuracy of using DEA to aid decision makers in their own setting. Given its non-parametric approach, ability to handle multiple inputs and outputs of different measurement units and previous research showing unique insight into similar problems, DEA provided a promising solution.

More specifically this research intends to:

- Create a new approach, based on DEA, for mathematically ranking DMUs from a list of competing candidates

- Develop a new extension to the theoretical proposition of using DEA as a pre-processor in MCDM problems where selection of a number of alternatives out of a larger population is required and expert opinion is not available.

- Demonstrate these theories by applying them to real-life data

4.1.2 Extend the use of DEA as part of a new a Multi Criteria Decision Making (MCDM) approach

- Combine MCDM and DEA theories for the selection of criteria weights and propose a theoretical framework for understanding DEA derived weights.

- Explore the relationship between DEA derived weights and Expert Opinion in MCDM. Specifically, looking at the weights defined using the new DEA based models and justifying the analogy of those weights as reflecting the consensus of how DMUs want to be evaluated.

- Develop a new methodology for applying DEA to MCDM problems that overcomes the recognized pitfalls as reported in the literature.

4.2 PROBLEMS WITH USING DEA FOR MCDM

There are two problems identified in the literature with the application of DEA to MCDM problems which this research addresses:

1. **Caution must be taken in the categorization of criteria into Inputs and Outputs [DOYL93].** The setup of a DEA model requires the specification of criteria to be an Input or Output. While it intuitively makes sense to have those criteria values you wish to minimize as Inputs and those you wish to maximize as Outputs, there is no concrete methodology for making this distinction and the results of the analysis are dependent on this step. Stewart also comments on this problem stating that while it is generally true that decision criteria can be

classified into Inputs and Outputs it is not always the case and this step may present “substantial difficulties” [STEW94].

2. **Perceived reliance on ‘strange ratios’ that must be defended to the DMUs being evaluated. [TOFA96].** Normal DEA creates a ratio of weighted outputs to weighted inputs, where the weights are calculated individually for each DMU. In the unconstrained case, it is possible that some outputs and/or inputs are assigned a weighting of zero when calculating the best score of that DMU. Therefore, it is possible that a DMU is scored and evaluated based on a ratio that mathematically makes sense but loses its intuitive meaning. For example and at the risk of getting ahead of ourselves a bit, the Power Plant Site Selection problem with which we deal in Section 5.1.3, could possibly place all of its weight on criteria 2 and 5. The resultant efficiency score is then reduced to the weighted ratio of megawatts generated per village evacuated. Doyle et. al address this concern pointing out that the ratio is dimensionless and should not be looked at in this fashion, that the resultant efficiency score is what should be interpreted and analysed [DOYL95B]. He does not address the issue that the analyst would have to explain the fact that a decision is being justified on such an illogical ratio to the DMUs themselves which is of paramount importance for the application space of this research.

The RC methodology presented next illustrates how to model MCDM problems, using DEA, while avoiding these problems. Standardizing the orientation of all criteria measures allows us to *push* all criteria to one side of the DEA model. This avoids the problem of ratios, and the problem of deciding inputs and outputs, at the same time. Using a one-sided model and standardizing the measures for criteria to the full zero to one scale allows us to compare expert weights to derived weights. Finally, new approaches to weight calculation embodied in the new OSD models that will be presented and explained in this Section build on the concept of cross-efficiencies, but extend them into the area of ranking.

4.3 THE RC METHODOLOGY (RCM)

Given the assumptions and limitations presented and discussed in Section 1.1, the RC Methodology (RCM) is summarized in Table 5 and presents the approach from original data collection through to final ranking.

4.3.1 Seven Steps of RCM

1	**COLLECT & ORIENT MEASURES** For each Alternative to be ranked, determine the criteria to be measured and collect the measurements. The key here is to orient all measures in a positive sense such that a preferred alternative will have a higher measure as compared to an alternative that is less preferred.
2	**NORMALIZE SCALES** For each criterion, we must now normalize the measurements so that they are scaled to the full zero to one interval [0,1]. This can be accomplished by dividing each measured value by the maximum possible value for that criterion.
3	**CALCULATE INDIVIDUAL WEIGHTS** For each Alternative, calculate an optimal distribution of weights from its perspective. In this thesis we present three of the possible mathematical programming based models for calculating these weights. The OSD-CCR model calculates weights that attempt to have an individual alternative 'rank first' when evaluated with those weights. The OSD-IP model calculates weights that attempt to have an individual alternative 'maximize its rank' when evaluated with those weights. The OSD-DA model calculates weights that attempt to have an individual alternative 'distinguish itself from average' when evaluated with those weights.
4	**STACK WEIGHTS** Stacking the weights from each alternative found in Step 3, component by component is the bridge between calculating weights from an individual or local perspective and moving into a group or global perspective.
5	**CALCULATE CONSENSUS WEIGHT VECTOR** The consensus weight vector is found by calculating the vector sum of all vectors from Step 4. The result is our Weight Importance Factor (WIF) which is interpreted to represent the consensus opinion of the alternatives being evaluated on how important they perceive each of the criteria to be when being scored and ranked.
6	**CALCULATE CONSENSUS SCORE** Taking the simple dot product of the WIF vector with the original criteria measures (combining Step 2 and Step 5) results in a single Weight Importance Derived Efficiency (WIDE) Score, between zero and one, for each alternative.
7	**RANK BY CONSENSUS** The results from Step 6 can now be used to rank order the alternatives.

Table 5 – The Seven Steps of Ranking by Consensus Methodology

4.3.2 New Methodology for solving MCDM problems with DEA

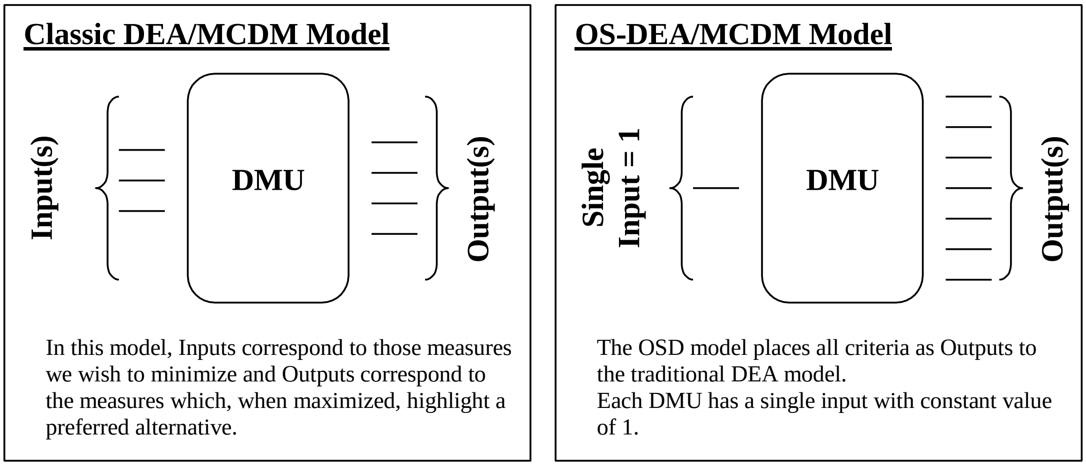


Figure 10 – Comparison of Classic DEA/MCDM Model and OS-DEA/MCDM Model

The basic premise of this new model for using DEA to analyze an MCDM problem involves dropping the notion that Inputs are to be minimized and Outputs are to be maximized. Instead, we would propose that the Outputs in the traditional DEA model represent *all* Criteria being used for the decision and one single Input, constant and identical for all DMUs is used to represent the common benchmark for analysis. In order to make this model work, the items that were typically assigned as inputs due to the requirement that they be minimized, need to be translated such that they now should be maximized. Of course, this in and of itself is not new. Others have approached DEA problems where they shifted either all inputs to be outputs or all outputs to be inputs (See Section 2.4.7). The major difference is that with RCM, the purpose for doing this is to standardize the decision space to allow for the creation of new ranking tools and approaches. Hence we are convinced that making this change will allow DEA technology to serve different goals than the efficiency measurement applications originally envisioned.

Again, looking ahead to the Power Plant example from Section 5.2, 'Villages to be Evacuated' was considered to be an input to the decision process since a lower score was more desirable. However, in this new model, we would propose that this variable be translated, so that the higher value is better. This does not change the meaning of the criterion; it is just a new way to code the value. What this permits a

decision maker to do however is to include this variable as an Output in the DEA model. One of the ways this translation can be performed is by taking the inverse (1/criteria value) and renormalizing the data [GREE87]. In the case where a criteria is coded based on an ordinal scale (i.e. 1 to 4), the data can be translated by inverting the scale where a score of 1 translates to a score of 4, a 2 becomes a 3 and vice versa (this method was demonstrated in [DOYL93]).

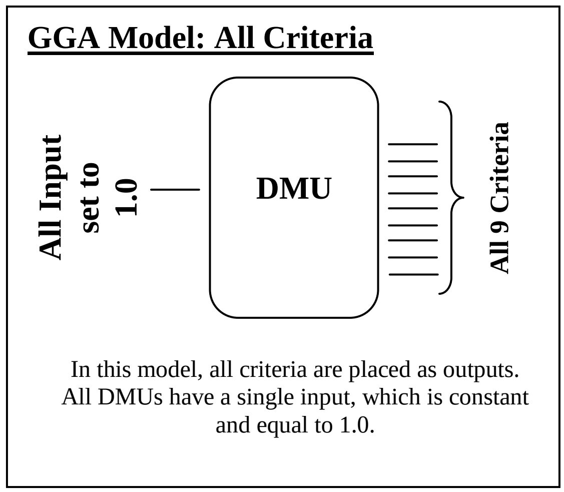


Figure 11 – Modeling GGA Data using OSD Models

When this procedure is performed for all measures that would be Inputs, we end up with a model that has all criteria listed as Outputs to be maximized, one single input that is set to 1.0 for each DMU, and a formulation that intuitively makes more sense when extending the analogy of self and peer appraisal in that the DMUs are now poised to assign weights to each criteria, taking into account trade-offs between each of those criteria, in line with what an expert would do in a classical MCDM problem. We call this new model a One-Side DEA model for MCDM problems or OS-DEA/MCDM (OSD for short). Figure 11 illustrates the OSD model for GGA Dataset from Section 5.0.

The results of such an analysis are intriguing and more insightful than the standard methodology for using DEA in an MCDM setting. We now have more information from which to draw conclusions, namely the weights assigned to each criterion by each DMU, as well as the efficiency score and the new Weight Importance Derived

Efficiency Score (WIDE Score) described in Section 4.3.4. First however, we will show how the OSD model can be derived from the standard DEA CCR models.

4.3.3 OSD-CCR Formulation Derivation

OSD-CCR Multiplier Form – From inspection

$$\begin{aligned} \max \quad & uY_o \\ \text{st} \quad & uY \le 1 \\ & u \ge 0 \end{aligned} \tag{4.1}$$

which is equivalent to:

$$\begin{aligned} & \max h_o = \sum_{r=1}^{s} u_r y_{ro} \\ & s.t. \quad \sum_{r=1}^{s} u_r y_{rj} \le 1 \quad j = 1,\dots,n \\ & \quad u_r \ge 0 \quad r = 1,\dots,s \end{aligned} \tag{4.2}$$

OSD-CCR Envelopment Form – From primal/dual properties of LP

$$\begin{aligned} \min \quad & \sum \lambda \\ \text{st} \quad & \lambda Y \ge Y_o \\ & \lambda \ge 0 \end{aligned} \tag{4.3}$$

which can also be expressed as:

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} \lambda_j \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j \cdot y_{rj} \ge y_{ro} \qquad r = 1,\dots,s \\ & \lambda_j \ge 0 \quad j = 1,\dots,n \end{aligned} \tag{4.4}$$

OSD-CCR Multiplier Form

To develop the OSD formulation, we start with the DEA-CCR primal formulation:

$$
\begin{aligned}
& \max h_o = \sum_{r=1}^{s} u_r y_{ro} \\
& s.t. \qquad \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \qquad -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} \le 0; \quad j = 1,\ldots,n \\
& \qquad u_r, v_i \ge 0 \quad r = 1,\ldots,s \;\; i = 1,\ldots,m
\end{aligned}
\tag{4.5}
$$

and let m=1 and x_{1j}=1 for all j's

$$
\begin{aligned}
& \max h_o = \sum_{r=1}^{s} u_r y_{ro} \\
& s.t. \qquad \sum_{i=1}^{1} v_i = 1 \\
& \qquad -\sum_{i=1}^{1} v_i + \sum_{r=1}^{s} u_r y_{rj} \le 0; \quad j = 1,\ldots,n \\
& \qquad u_r, v_i \ge 0 \quad r = 1,\ldots,s \;\; i = 1,\ldots,m
\end{aligned}
\tag{4.6}
$$

The first constraint forces the sum of v_i's to be equal to 1. Therefore, substituting into the second constraint:

$$
\begin{aligned}
& \max h_o = \sum_{r=1}^{s} u_r y_{ro} \\
& s.t. \qquad v_1 = 1 \\
& \qquad -1 + \sum_{r=1}^{s} u_r y_{rj} \le 0; \quad j = 1,\ldots,n \\
& \qquad u_r \ge 0 \quad r = 1,\ldots,s
\end{aligned}
\tag{4.7}
$$

which then simplifies to:

$$
\begin{aligned}
& \max h_o = \sum_{r=1}^{s} u_r y_{ro} \\
& s.t. \qquad v_1 = 1 \\
& \qquad \sum_{r=1}^{s} u_r y_{rj} \le 1; \quad j = 1,\ldots,n \\
& \qquad u_r \ge 0 \quad r = 1,\ldots,s
\end{aligned}
\tag{4.8}
$$

This formulation can be further reduced by noticing that the v's no longer play a role in the formulation, giving:

$$
\begin{aligned}
& \max h_o = \sum_{r=1}^{s} u_r y_{ro} \\
& s.t. \qquad \sum_{r=1}^{s} u_r y_{rj} \le 1; \quad j = 1,\dots,n \\
& \qquad\qquad u_r \ge 0 \quad r = 1,\dots,s
\end{aligned}
\tag{4.9}
$$

which is equivalent to the OSD-CCR Multiplier formulation.

OSD-CCR Envelopment Form

The envelopment form of the OSD formulation can be derived in a similar manner.

$$
\begin{aligned}
& \min \quad \theta \\
& s.t. \quad \theta \cdot x_{io} - \sum_{j=1}^{n} \lambda_j \cdot x_{ij} \ge 0 \qquad i = 1,\dots,m \\
& \qquad \sum_{j=1}^{n} \lambda_j \cdot y_{rj} \ge y_{ro} \qquad r = 1,\dots,s \\
& \qquad \lambda_j \ge 0 \quad j = 1,\dots,n
\end{aligned}
\tag{4.10}
$$

Starting with the standard form of the CCR dual, we let m=1 and x_{1j}=1 for all j's

$$
\begin{aligned}
& \min \quad \theta \\
& s.t. \quad \theta - \sum_{j=1}^{n} \lambda_j \ge 0 \\
& \qquad \sum_{j=1}^{n} \lambda_j \cdot y_{rj} \ge y_{ro} \qquad r = 1,\dots,s \\
& \qquad \lambda_j \ge 0 \qquad j = 1,\dots,n
\end{aligned}
\tag{4.11}
$$

From here, we notice that the first constraint forces θ to be greater than or equal to the sum of all λs. Since there are no other constraints on θ and it is to be minimized then the net effect is to have θ equal to the sum of all λs at the optimal point, allowing us to simplify to:

$$
\begin{aligned}
&\min \quad \theta \\
&s.t. \quad \theta = \sum_{j=1}^{n} \lambda_j \\
&\qquad \sum_{j=1}^{n} \lambda_j \cdot y_{rj} \geq y_{ro} \qquad r = 1, \ldots, s \\
&\qquad \lambda_j \geq 0 \qquad j = 1, \ldots, n
\end{aligned}
\tag{4.12}
$$

Therefore, we can remove the θ variable:

$$
\begin{aligned}
&\min \quad \sum_{j=1}^{n} \lambda_j \\
&s.t. \quad \sum_{j=1}^{n} \lambda_j \cdot y_{rj} \geq y_{ro} \qquad r = 1, \ldots, s \\
&\qquad \lambda_j \geq 0 \qquad j = 1, \ldots, n
\end{aligned}
\tag{4.13}
$$

Which is equivalent to the OSD-CCR Envelopment formulation.

4.3.4 WIGs, WIFs and WIDE Scores

The main shift in thinking embodied by this new methodology is the analogy of the

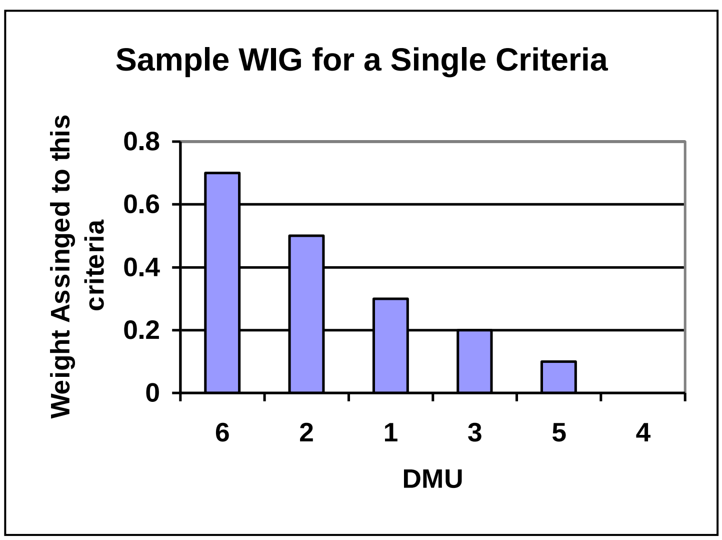


Figure 12 – Sample WIG for a Single Criterion[1]

geometric mean of weight vectors assigned to a criterion by DEA can be considered to represent the consensus of all DMUs on how important they perceive that criteria

to be, from their perspective, when being compared to all other DMUs. This was not possible with the models and methodologies from the literature due to the fact that some criteria were considered inputs and others considered outputs. This prevented the direct comparison of Expert and DEA assigned weights and did not allow for the interpretation of those weights as the importance a DMU places on each criteria.

Three new terms are defined for working with one-sided DEA models. The first term is a Weight Importance Graph (WIG), which is an ordered distribution of the weight assigned by *each* DMU to *one* criterion. If there were six criteria as in the Stewart example, we would have six WIGs. Figure 12 is an illustration of a sample WIG and does not reflect actual data from this research. The Y axis of the graph corresponds to the observed weight assigned to a single criterion and each point on the X axis refers to a single DMU. The data points can be sorted in descending order as in Figure 12 to help visualize and compare the graphs across multiple criteria.

The significance of these graphs comes from the calculation of the area under the resultant distribution when the values of the weights are plotted. This is a measure of how much weight, across all DMUs, is placed on that criteria while making each DMU look as good as possible. In the fictitious example illustrated in Figure 12, the resultant area under the graph would be 1.8. Keep in mind that there would be an equivalent WIG for each of the criterion in the problem and the areas under each of those graphs would need to be calculated.[2]

Normalizing the values of the area under *all* graphs gives one vector called a Weight Importance Factor (WIF). The WIF will have as many elements as there are WIGs and simply presents the proportion, on average, that a DMU attributes to each corresponding criteria. The WIF is the link between DEA derived weights and MCDM expert elicited weights. The WIF vector will have elements that sum to 1 and represent a common set of weights that can be applied to each DMU but instead of this vector being determined by experts as in an MCDM problem, they are determined by the DMUs themselves in an RC problem.

[1] Note: This graph is presented for illustrative purposes only and does not correspond to actual results.

$$\overline{WIF} = \left[\frac{WIG_1}{\sum_{r=1}^{s} WIG_r}, \frac{WIG_2}{\sum_{r=1}^{s} WIG_r}, \ldots, \frac{WIG_s}{\sum_{r=1}^{s} WIG_r} \right] \tag{4.14}$$

where WIF = Weight Importance Factor Vector

WIG_r = Area under distribution of Weight Importance Graph r

s = total number of criteria

Equation (4.14) presents the simplified formulation for the weight importance factor as calculated from the graphical representation of individual DMUs' optimal weights depicted in the weight importance graphs. In this formulation, each DMU is allocated an equal influence in determining the consensus weight vector.

$$\overline{WIF} = \overline{\left[\sum_{j=1}^{n} a_j \overline{u_j} \right]} = \overline{\left[a_1 u_{11} + a_1 u_{12} + \ldots + a_1 u_{1s}, \ldots, a_n u_{n1} + a_n u_{n2} + \ldots + a_n u_{ns} \right]} \tag{4.15}$$

where WIF = Weight Importance Factor Vector

a_j = scalar multiple of influence (default value = 1.0)

s = number of criteria

n = number of alternatives

The general form of the weight importance factor is presented in Equation (4.15). Here, we introduce a scalar variable 'a' to allow for unequal influence between different alternatives in contributing to the consensus weight vector. This general form is more flexible in that under certain contexts, providing more influence to a subset of alternatives could be beneficial.

$$Expert\ Adjusted\ \overline{WIF} = \mathrm{b} \cdot \overline{(\text{Expert Weights})} + (1\text{-b}) \cdot \overline{WIF} \tag{4.16}$$

where EA-WIF = Expert Adjusted Weight Importance Factor

b = scalar between 0 and 1 representing proportion influence between expert and alternatives

Finally, generalizing to the case where a decision maker would like to balance their expert opinion to the perspective generated by the consensus of the alternatives, we

[2] Note: The area under a WIG is simply the sum of the weights assigned by all DMUs for a single criteria.

have Equation (4.16) which introduces the scalar ‘b’ to account for relative influence between the two.

From the WIF vector or EA-WIF vector, we can then calculate Weight Importance Derived Efficiency Scores (WIDE Scores). These scores are calculated by taking the sum product of the vector of normalized criteria values (Outputs in this model) by the WIF vector resulting in a single score between 0 and 1. These scores can be interpreted as the rating given to a particular DMU when evaluated against the weights that the whole population of DMUs conceded to.

$$WIDE\,Score_i = \overline{WIF} \bullet \overline{Outputs_i} \tag{4.17}$$

where WIF = Weight Importance Factor Vector
$Outputs_i$ = Vector of normalized criteria values for DMU i

The WIDE scores are analogous to cross-efficiency scores and provide a method for ranking DMUs without having to calculate an n x n cross-efficiency matrix. The WIDE scores are also analogous to a score that one would calculate in a typical MCDM sense where the sum product of the criteria and common weight vector give an overall score.

4.3.5 Advantages and Benefits of OSD-CCR

The OSD-CCR model and RC methodology address the shortcomings associated with using DEA on MCDM problems identified in the literature and represent a new approach to tackling these problems.

1) Caution must be taken in the categorization of criteria into Inputs and Outputs [DOYL93].

The OSD methodology does not require the analyst or decision maker to have to make a judgment whether a criteria is an input or output. All criteria are outputs and only the scale of the data needs to be translated to reflect this.

2) Perceived reliance on 'strange ratios' that must be defended to the DMUs being evaluated. [TOFA96].

With all criteria placed on one side of the DEA model, we allow for trade-offs among criteria to be easily seen. A decision maker can now easily justify to the DMUs how they are being scored in that the WIFs and WIDE scores are calculated using weights derived by DEA that show them in the best possible light, while maintaining feasibility.

4.4 PROBLEMS WITH OSD-CCR

The OSD-CCR model is based on the fact that the DEA approach chooses weights for an individual DMU in a manner that makes the DMU as 'attractive as possible'. Other language used to describe the basic DEA mechanism is to choose weights to make a DMU 'appear in the best possible light' or to choose weights to 'maximize the efficiency score'. Each of these descriptions are valid in the context of efficiency analysis, where the efficiency score is the single value which we are trying to optimize while maintaining feasibility of the whole system.

In the ranking context however, we find that the efficiency definition and more specifically, the efficiency score may become barriers to fully realizing the potential of advanced linear programming techniques for choosing weights for any particular DMU. Looking at the DEA mechanism with the goal of ranking in mind, we can now define a new description of what the mathematics is saying. Starting with the OSD formulation from Section 4.3, Equation (4.1) (repeated here for convenience) we can see that the objective function attempts to maximize the weighted score of the DMU under investigation by searching all possible weight vectors while maintaining that no DMU's score is greater than 1.0.

$$\begin{aligned} \max \quad & w_o = uY_o \\ \text{s.t.} \quad & uY \le 1 \\ & u \ge 0 \end{aligned}$$

Another way to look at the same formulation is to remove the concept of efficiency score and look solely at rank. From this perspective, the same program can be seen to 'choose weights to rank first'. Said another way, the program will scan the entire production possibility set for a weight vector which ranks the DMU under investigation as first overall (while allowing for ties). Therefore, if the optimal value of the objective function is 1.0, then there exists a set of weights for which the DMU under investigation ranks first overall.

This then begs the question, what if the program cannot find a set of weights to enable a DMU to rank first? To address this issue we recall that we want to remain in a ranking context and realize that any DMU that does not rank first, by definition ranks second. What this means at the highest level is that using the standard DEA mechanism for ranking only allows us to rank DMUs into two positions, those tied for first, and those tied for second. This leads to an interesting dilemma. While for those DMUs that are deemed DEA efficient have had weights chosen for them with the goal of ranking them first, the DMUs that are not efficient are simply assigned weights based on a radial projection from the origin to the enveloping frontier. Thus, for the purpose of consensus weighting of selection criteria, we can argue to those efficient DMUs that their contributed weights *maximized their rank* but we cannot say the same for the inefficient DMUs. For situations where it is appropriate for a subset of alternatives to decide on a set of weights for the entire population, the OSD-CCR formulation may be the most applicable. However, when only a subset of alternatives will benefit from the application of a consensus weight vector, taking the aggregation of all DMUs' OSD-CCR weights to represent the consensus may not be the best choice as not all DMUs are free to choose weights to maximize their rank.

It was the identification of this property that has lead to a new Integer Programming formulation that does answer the question – 'choose weights to maximize rank' for *all* DMUs. The ability to connect the choice of weights directly to the concept of rank while bypassing the concept of an efficiency score allows us to calculate an optimal set of weights for each DMU, stack those weights for consensus, and rank DMUs in a more fair manner.

4.5 OSD-IP FORMULATION

The main approach presented in this thesis can be summarized as: Given the assumptions and limitations presented in Section 1.1, and given a finite set of DMUs and corresponding scores on a finite set of criteria, calculate a set of weights for each DMU that maximizes their rank in the population. Then, stack those weights to obtain one common set of weights (used to represent the consensus) and then use the common set of weights to rank all DMUs.

The integer program which calculates an optimal set of weights to maximize rank is found in Equation (4.18).

$$\begin{aligned}
&\min \Omega = \sum_{j=1}^{n} z_j \\
&\text{st} \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{s} u_r y_{ro} - z_j \le 0 \quad j = 1,\dots,n \\
&\qquad \sum_{r=1}^{s} u_r = 1 \\
&\qquad u_r > 0 \\
&\qquad z_j = 0 \text{ or } 1
\end{aligned} \tag{4.18}$$

The OSD-IP formulation acts as a counter where the optimal objective function value $\Omega*$ provides a count of those DMUs that rank above the DMU under investigation. The mechanics of the program works in an analogous manner to the calculation of the envelopment surface in a standard DEA program but does not restrict the defined hyperplanes to those DMUs on the frontier. Instead, the program creates a localized hyperplane through each DMU that minimizes the number of DMUs *above* DMU_o. The reason this works in the OSD context but not in a traditional DEA context is that the meaning of *above* makes sense both mathematically and intuitively in OSD when all criteria are oriented in such a way that a higher value would indicate a better performer.

The first constraint of the program, $\sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{s} u_r y_{ro} - z_j \le 0 \quad j = 1,...,n$, can be best understood by taking each term individually. The first term, $\sum_{r=1}^{s} u_r y_{rj}$ corresponds to the weighted score of DMU_j. The second term, $\sum_{r=1}^{s} u_r y_{ro}$ corresponds to the weighted score of the DMU under investigation (DMU_o). If the difference between these two terms is negative, then this implies that DMU_o will rank higher than DMU_j and the constraint is satisfied by z_j being set to zero or being set to one, but since the objective function is attempting to minimize the sum of all z_j's, it will receive a zero value.

If on the other hand, DMU_j scored higher than DMU_o, then the value of z_j would have to be set to one in order to satisfy the first constraint. This increases the count of the total number of DMUs that score higher than DMU_o.

The second constraint, $\sum_{r=1}^{s} u_r = 1$ fixes the sum of the weights to the value of 1.0 to set a benchmark for comparison to all other DMUs. The actual value is arbitrary as long as it is constant for all. The third constraint, $u_r > 0$ ensures that all weights are positive and the final constraint imposes an integer restriction on the *z* variable.

Taken together, this program calculates a set of optimal weights u_r* for each DMU that maximizes its rank, allowing for the consensus weighting of selection criteria. In situations where there is a limited number of DMUs to select, the weights found using the OSD-IP formulation is appropriate as it attempts to find weights to have a DMU rank in the top positions.

4.6 OSD-DA FORMULATION

There may exist situations where the weights calculated using OSD-IP are not the weights that a DM would choose to use in evaluating and ranking the DMUs. These situations would arise when a set of weights are to be applied to all DMUs without

the caveat that only a limited number will be selected. Policy decisions tend to fall into this category but also the case of making funding decisions for primary research where the benefits affect the entire population. In these situations, absolute fairness may become secondary to the uniqueness of the research being conducted. An alternative OSD formulation can be created that *chooses weights for each DMU to distinguish it from the average DMU*. In this way, distinctness or uniqueness of a DMU is reflected in the choice of optimal weights. The corresponding linear program which accomplishes this goal is as follows:

$$\begin{aligned} \min \Phi &= \sum_{j=1}^{n} \sum_{r=1}^{s} u_r y_{rj} \\ \text{st.} \quad & \sum_{r=1}^{s} u_r y_{ro} = 1 \\ & u_r > 0 \end{aligned} \tag{4.19}$$

Simply stated, Equation (4.19) chooses weights, u_r, to minimize the sum of all the DMUs' scores while fixing the score of DMU_o at an arbitrary constant (set to 1 for simplicity). This has the effect of isolating that direction (i.e. set of weights) which maximizes the distinction between DMU_o and the average DMU. For long term planning and policy making decisions, this approach may be superior to the OSD-IP approach in that it highlights and rewards DMUs for differentiating themselves.

4.7 OSD EXTENSIONS

Other programs can similarly be made. Due to the fact that all dimensions of the problem are oriented in such a way that a higher value indicates a better performer, we now have an intuitive sense of *direction* that was suspect in the traditional DEA context. This allows us to create programs that can identify the direction vector (set of weights) for the whole population of DMUs which either minimizes or maximizes the average score, giving a sense of where the DMUs are competitive and non-competitive.

$$\begin{aligned} \max \quad & \sum_{j=1}^{n}\sum_{r=1}^{s} u_r y_{rj} \\ \text{st.} \quad & \sum_{r=1}^{s} u_r = 1 \\ & u_r > 0 \end{aligned} \tag{4.20}$$

Equation (4.20) will find weights that maximize the global score and correspondingly, equation (4.21) will find those weights, which minimizes the global score.

$$\begin{aligned} \min \quad & \sum_{j=1}^{n}\sum_{r=1}^{s} u_r y_{rj} \\ \text{st.} \quad & \sum_{r=1}^{s} u_r = 1 \\ & u_r > 0 \end{aligned} \tag{4.21}$$

4.8 AGGREGATION OF OSD WEIGHTS

Steps 4 and 5 of the RC methodology are concerned with the *stacking* of weights to calculate the consensus weight vector. The output of Step 3 is a collection of weight vectors from each alternative's perspective that need to be aggregated. This is accomplished through the simple vector addition of individual weight vectors.

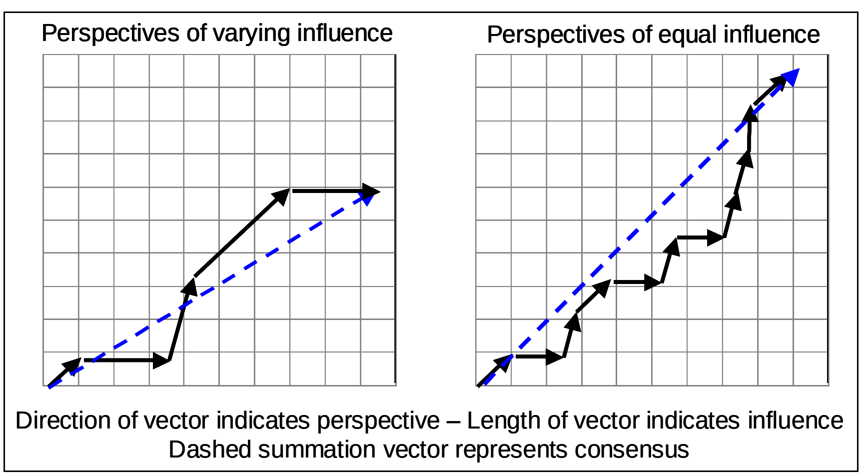


Figure 13 – Aggregating perspectives in RCM

For the analysis and examples presented next in Section 5.0, each weight vector is normalized such that its magnitude is equal to 1.0 as we are only considering the direction implied by each weight vector and we assign an equal amount of influence to each alternative when calculating our WIF vector. This does not have to be the case however, and as can be seen in Figure 13, adjusting the magnitude of individual weight vectors allows us to model varying influence in determining the final WIF. Equation (4.15) formalizes this property.

Another advantage of RCM that becomes apparent from Figure 13 is the ease with which expert opinion can be incorporated into the model. Experts can inject their own weight vectors, decide on the magnitudes of individual weight vectors and exclude other weight vectors when appropriate. Equation (4.16) formalizes this concept to the case where Expert opinion is included in the determination of the final WIF. It is the concept of direction that separates the RC methodology from other approaches and allows us to not only rank alternatives, but also provide feedback on which direction(s) to focus on to improve in rank.

4.9 SUMMARY

The programs and methods contained in this section are by no means an exhaustive presentation of those that can be created using the RC/OSD approach. In Section 6.0 we will make recommendations for future directions and enhancements to the models and methods presented here. Section 5.0 will explore four relevant applications to demonstrate the application of the RC methodology and the use of the OSD models.

5.0 APPLICATIONS OF RCM

"We all agree that your theory is crazy, but is it crazy enough? – Niels Bohr

This Section illustrates the application of RCM to four different ranking problems, two site selection problems, one funding problem and a two-dimensional simulated problem. For each, the seven steps were applied resulting in the determination of one or more sets of consensus weights. These weights were then used to score and rank the alternatives. Additionally, for the two-dimensional problem, illustrations are presented to better explain the mechanics of the models and methods employed. See Appendices A, B, C and D for Lindo programs, results and summary sheets for each respective application.

5.1 THE DATASETS

Dataset	Source	Variables
STEW92	• Nuclear Power Plant Site Selection from the literature	• Manpower, Construc.Cost, Mainten.Cost, Villages Evacuated, Power Generated, Safety Level
GGA	• Undisclosed	• 9 criteria – mapping undisclosed
COOP00	• Capital City site selection from Cooper, Seiford & Tone 2000 pp 169-174	• 7 criteria – mapping undisclosed
PARA04	• Working paper	• 2 criteria, 10 alternatives, simulated data

Table 6 – Summary of Available Data Sources

Four interesting and relevant datasets were secured for this research. The first dataset is taken from the literature and involves the selection of a site for a Nuclear Power Plant. The second dataset was donated by a large Government Granting Agency (GGA) and consist of 200 randomly chosen applications for one of their national competitions. The third is taken from [COOP00] and analyses the decision to select the winner out of seven candidate sites for the location of a new capital city. Lastly,

a simulated two-dimensional dataset was created, consisting of ten alternatives evaluated on two criteria for illustrative purposes.

5.1.1 Dataset 1 – Power Plant Site Selection

Stewart presents a dataset for the problem of selecting a site for a Nuclear Power Plant as his standard dataset for comparing different MCDM techniques [STEW92]. This dataset has been further used in [STEW94], [DOYL93] and [TOFA96] to validate extensions to DEA/MCDM theory.

DMU	Manpower (Minimize)	Power Generated (Maximize)	Construc. Cost (Minimize)	Mainten. Cost (Minimize)	Villages Evacuated (Minimize)	Safety Level (Maximize)
Italy	80	90	600	54	8	5
Belgium	65	58	200	97	1	1
Germany	83	60	400	72	4	7
UK	40	80	1000	75	7	10
Portugal	52	72	600	20	3	8
France	94	96	700	36	5	6

Table 7 – Raw Data for Power Plant Site Selection

This dataset (reproduced in Table 7) contains 6 DMUs (which correspond to possible locations for the power plant) and 6 criteria (with scores) and will be used to compare and contrast results of this research with results and conclusions from the literature.

5.1.2 Dataset 2 – Shortlisting Research Grant Proposals for GGA

A dataset containing expert reviewed scores on nine criteria was provided by the GGA. The dataset included 200 randomly drawn applications, each record containing scores on each of the nine criteria as well as a scaled dollar amount, which corresponded to the actual funds requested. Expert recommendations to fund or not fund the project were also provided. The mapping of criteria the GGA use to evaluate and rank individual proposals was not disclosed.

The GGA provided the data with certain limitations and omissions to protect the privacy of applicants and avoid any conflict of interest. While the omissions restrict the discussion on the specifics of the GGA application process and selection criteria, it does not impact the objectives of this research.

5.1.3 Dataset 3 – Capital City Site Selection

A simplified version of the actual data used for the real application of selecting from a set of candidate sites for a new capital city is examined here.

Alternative	C1	C2	C3	C4
A	5	10	3	5
B	7	10	3	10
C	8	7	10	5
D	4	8	3	10
E	9	4	4	2
F	10	5	10	3
G	4	7	7	8

Table 8 – Raw Data for Capital City Site Selection

The real problem involved nine candidate sites and many diverse criteria including distance to major cities, safety indexes regarding earthquakes and volcanoes, access to international airport, ease of land acquisition, landscape, water supply, matters of historical associations etc.

While expert opinion was also available for this problem in the form of weights for each criterion, for demonstration they are not included in our analysis. Comparison however between the OSD derived weights and expert derived weights is made possible due to the one-sided nature of our solution approach.

5.1.4 Dataset 4 – Two-Dimensional Illustrative Example

A simulated dataset was created consisting of ten alternatives evaluated on two criteria for the purposes of graphically illustrating the RC methodology and the three OSD models. Table 9 presents the raw data.

Label	Alternative	Criteria 1	Criteria 2
A	Orange	1	8
B	Violet	2	5
C	Blue	3	8
D	Yellow	4	4
E	Cyan	4	2
F	Black	5	7
G	Red	6	5
H	White	7	7
I	Green	7	4
J	Grey	8	2

Table 9 – 2-D Example Raw Data

5.2 POWER PLANT DATA RESULTS

5.2.1 Summary of Results

Table 10 summarizes and compares the results obtained from the standard method of analysing this problem using the methodology from the literature and the new RC methodology, where weights are chosen using the OSD-CCR model. A score of 1.0 was found for all DMUs except Italy with this model, which is consistent with the DEA results obtained with the normal DEA/MCDM model.

DMU	OSD-CCR Score (Rank)	WIDE Score (Rank)	Classical DEA/MCDM Score (Rank)	Aggressive Cross-Efficiency Score (Rank)	Standard Cross-Efficiency Score (Rank)
Italy	**0.989 (2)**	0.566 **(5)**	**1.000 (1)**	0.503 **(5)**	0.799 (3)
Belgium	**1.000 (1)**	**0.630 (1)**	**1.000 (1)**	0.465 **(6)**	0.741 (5)
Germany	**1.000 (1)**	0.578 **(4)**	**1.000 (1)**	0.530 **(4)**	0.702 (6)
UK	**1.000 (1)**	**0.630 (1)**	**1.000 (1)**	0.542 **(3)**	0.854 (2)
Portugal	**1.000 (1)**	0.611 **(2)**	**1.000 (1)**	**0.799 (1)**	**0.935** (1)
France	**1.000 (1)**	0.589 **(3)**	**1.000 (1)**	0.582 **(2)**	0.777 (4)

Table 10 – Comparison of OSD Results and Classical DEA/MCDM Results for Power Plant Data

It is not possible to rank the DMUs simply from their efficiency scores in both cases. The literature then goes on to calculate cross-efficiencies, both standard and aggressive formulations that immediately show Portugal to be the winner. In contrast, the calculated WIDE score identifies the UK and Belgium as tied for first place. Using the approaches from the literature, Belgium ranks near the bottom of the list for this problem. The source of difference in rank for Belgium is easy to see when we analyse the WIGs and WIF for this problem.

The six WIGs corresponding to the six decision criteria are presented in Figure 14. These graphs provide a visual representation of how important each DMU treats each of the criteria. Looking at the first graph in Figure 14, we see that the Manpower criterion was used in calculating an efficiency score by two of the DMUs (Italy and Belgium). Each of these DMUs attributed an equal amount of weight to this criterion.

Other interesting observations include the fact that no DMU used ‘Villages Evacuated’ when attempting to make itself look ‘as good as possible’. This is the

criteria where Belgium shines but the consensus of DMUs do not attribute any importance to this criteria thus contributing to Belgium's resultant low ranking.

Weight Importance Graphs for Power Plant Application

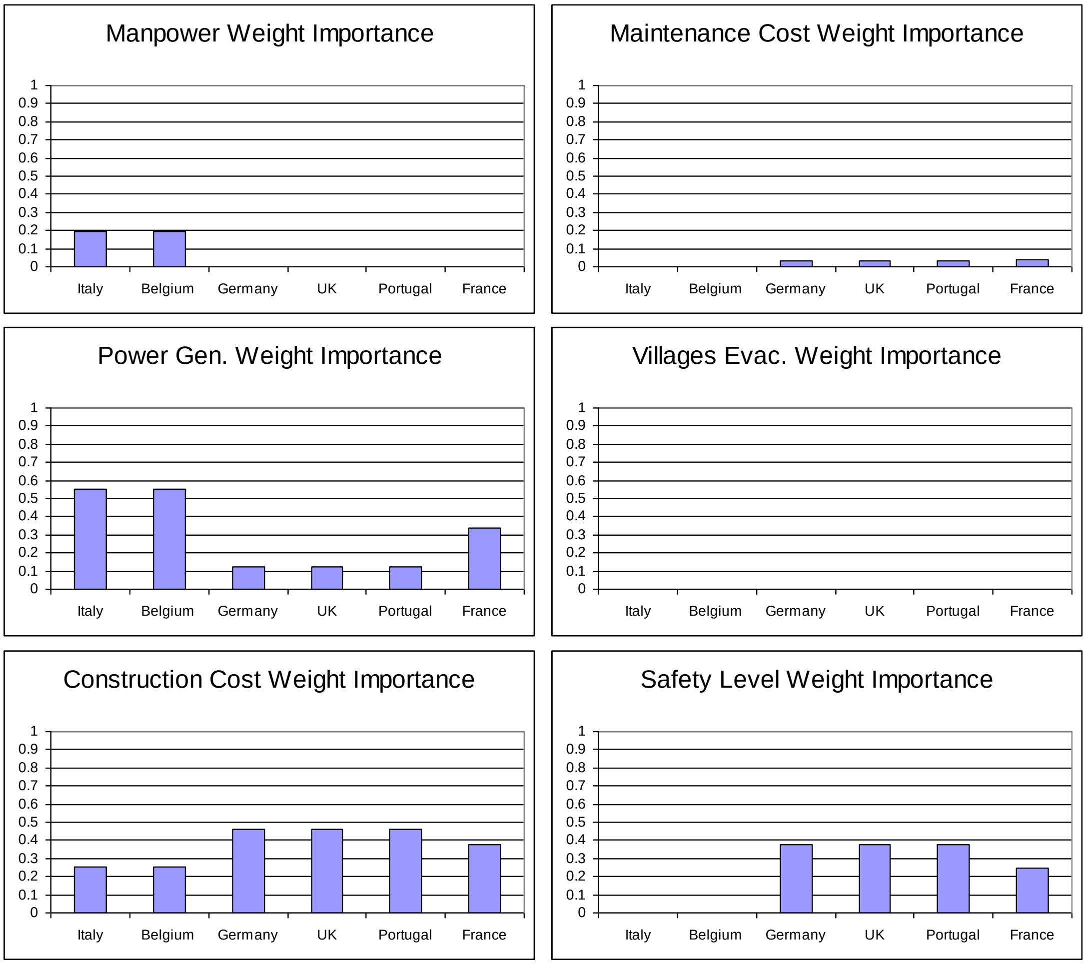


Figure 14 – Weight Importance Graphs for Each of the Six Criteria from the Power Plant Problem

Weight Importance Factors

The area under each WIG is approximately 0.4, 1.8, 2.3, 0.1, 0.0 and 1.4 respectively which when normalized gives a WIF of [7%, 30%, 38%, 2%, 0%, 23%]. It follows then that the consensus of the DMUs see Criteria 2, 3 and 6 as the most important and Criteria 1, 4 and 5 are almost negligible when, from the DMUs perspective, one is trying to compare and score each DMU.

WIDE Scores

The WIDE scores from Table 10 were calculated using the WIF in the previous section and taking the sum product with the original criteria values for each DMU.

5.2.2 Discussion

Converting this problem to an OSD problem did not change the overall ranking of DMUs based solely on their efficiency scores. The output from applying the OSD-CCR model is not only an efficiency score which we can use to compare DMUs but also weights for each criteria, which when stacked provides a unique perspective on what the DMUs perceive as the most important criteria.

The WIF can be used to compare expert assigned weights in a normal MCDM setting to the consensus opinion of the DMUs from an OSD sense. The reason for doing this is that the weights found from the OSD models are difficult to criticize by the DMUs as they are calculated to make the DMUs look as good as possible (while maintaining feasibility). While this may not be critical when picking a site for a Power Plant, it would be important for the problem of picking the winner(s) of a competition.

Overall, for this dataset, we can see that the new methodology and the normal DEA/MCDM methodology, at a minimum, provide the same information as far as overall efficiency score. The OSD model also avoids the identified pitfalls and actually provides more insightful information that is particularly useful to a decision maker. From this new methodology we can say that the DMUs have a strict preference on the criteria they would like to be judged against and a decision maker can then use this as a starting point for adding weight bounds if deemed necessary.

It is the WIF (the distribution of weight across criteria from the perspective of the DMUs) that is one of the major contributions of this work. The normal DEA/MCDM methodology could not provide this information because the weights were created in a ratio context (i.e. Outputs/Inputs). Direct trade-offs among the criteria were not easy to see or discuss because weight bounds had to be included as

ratios as opposed to absolute values, OSD avoids this difficulty and opens the door to direct communication between a DM and evaluated DMUs.

The application of this new methodology to the Stewart dataset has provided some interesting results but the inference and meaning of these results are limited in an MCDM problem where fairness, from the perspective of the DMUs, is not critical to the acceptance of the methodology. This is definitely the case for the GGA dataset examined next.

5.3 GGA DATA RESULTS

5.3.1 Summary of Results

The GGA dataset included 200 DMUs (applications for funding) of which 120 received a recommendation for funding by the GGA and 80 did not. The OSD-CCR model was run on this data and the results are summarized in Table 11 and Table 12.

Efficiency Score	Count of those DMUs that were funded	Count of those DMUs that were NOT funded
w_o= 1.0	55	5
0.75 < w_o < 1.0	44	7
w_o = 0.75	21	51
0.5 < w_o < 0.75	0	15
w_o <= 0.5	0	2
Total	**120**	**80**

Table 11 – Summary of Normal DEA Efficiency Scores for GGA Data

WIDE Score	Count of those DMUs that were funded	Count of those DMUs that were NOT funded
$WIDE_i$= 1.0	0	0
0.75 < $WIDE_i$ < 1.0	42	0
$WIDE_i$ = 0.75	6	0
0.70 < $WIDE_i$ < 0.75	19	1
0.65 < $WIDE_i$ < 0.70	32	4
0.60 < $WIDE_i$ <= 0.65	13	18
0.55 < $WIDE_i$ <= 0.60	7	17
0.50 < $WIDE_i$ <= 0.55	1	18
$WIDE_i$ <= 0.50	0	27
Total	**120**	**80**

Table 12 – Summary of WIDE Scores for GGA Data

Referring to Table 11, we can see that the standard efficiency score (optimal objective function value for each DMU) is a good predictor of whether a DMU will

receive funding or not. All DMUs with a score greater than 0.75 received funding by the GGA however at the exact score of 0.75, it is hard to distinguish between those DMUs that did receive funding from those that did not. In fact, almost 38% of DMUs obtained a score of 0.75 and 70% of those did not receive funding.

The WIDE score was much more effective at separating the funded DMUs from the not-funded DMUs. The WIDE score is calculated using the consensus weights found using the OSD-CCR model. In fact, all funded DMUs scored greater than 0.5. On the other hand, all of the non-funded DMUs scored less than 0.71.

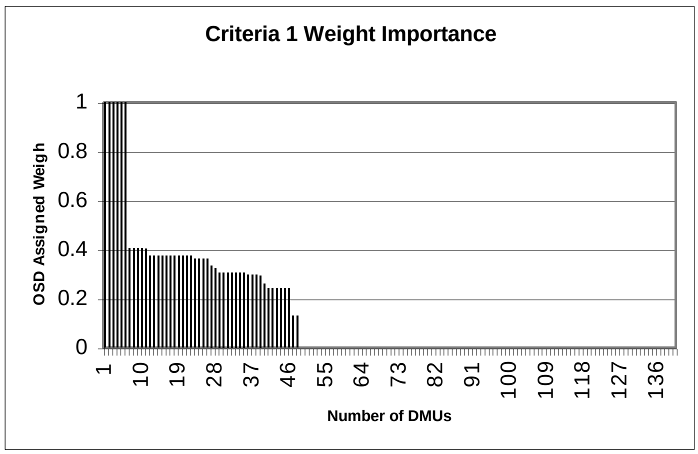


Figure 15 – Weight Importance Graph for Criteria 1 from GGA Dataset

One of the nine WIGs from this problem is presented in Figure 15. This graph illustrates the amount of weight each DMU assigns to Criteria 1 and is presented in a descending sorted order view. That is to say that the X axis does not correspond to a particular DMU but all inefficient DMUs are represented. By sorting the weights in this manner, we are able to quickly gain new insight into the problem. For example, it can easily be seen that 6 of the DMUs assigned a weight of 1.0 to this criteria, which accounts for all the possible weight it could assign across all criteria. In total, 47 of the DMUs (25%) placed some weight on this criteria and the remaining assigned zero weight. The total area under the curve is 19.5.

The area under each of the remaining eight WIGs presented in Figure 16 were found to be approximately 5.3, 19.8, 17.4, 15.4, 12, 7, 22.7 and 20.9 respectively.

Weight Importance Graphs for GGA

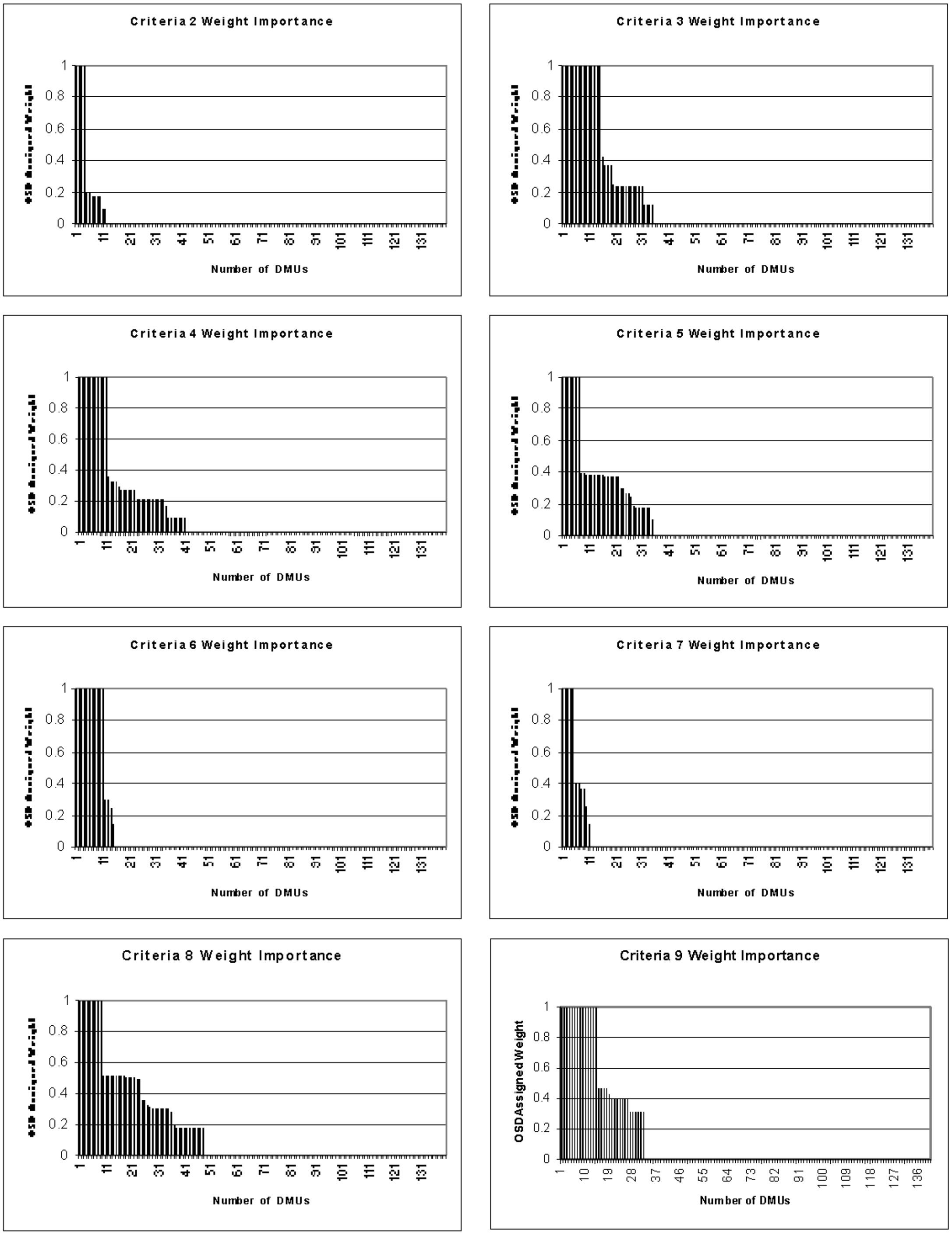


Figure 16 – Weight Importance Graphs for remaining 8 GGA Criteria

Note: The X Axis for each graph is a count of the number of observations and the Y Axis is the amount of weight each DMU assigned to a particular criteria. The area under each graph, when normalized, with each other graph gives the Weight Importance Factor for this problem.

Dimensions for these values are not needed because each of the criteria values have been normalized prior to the application of the model which allows for the comparison and discussion of these values in absolute terms. These values can be normalized to obtain the WIF for this problem.

Weight Importance Factors

The WIF for this problem is [14%, 4%, 14%, 12%, 11%, 9%, 5%, 16%, 15%] corresponding to the nine criteria for this problem. When this vector is applied to the criteria values for each DMU, we end up with a new WIDE score for each DMU, which can be used to rank the DMUs for selection.

WIDE Scores

The WIDE scores were calculated by taking the sum product of the WIF vector from the previous step and the original criteria scores for each DMU. The WIDE scores were found to correlate highly with OSD-CCR Efficiency scores (0.85). Figure 17 is a plot of WIDE Scores versus Efficiency Scores and shows how the WIDE Scores can be used to rank DMUs. There was a large population of DMUs with Efficiency scores of 1.0 and 0.75 however those same DMUs had different WIDE scores.

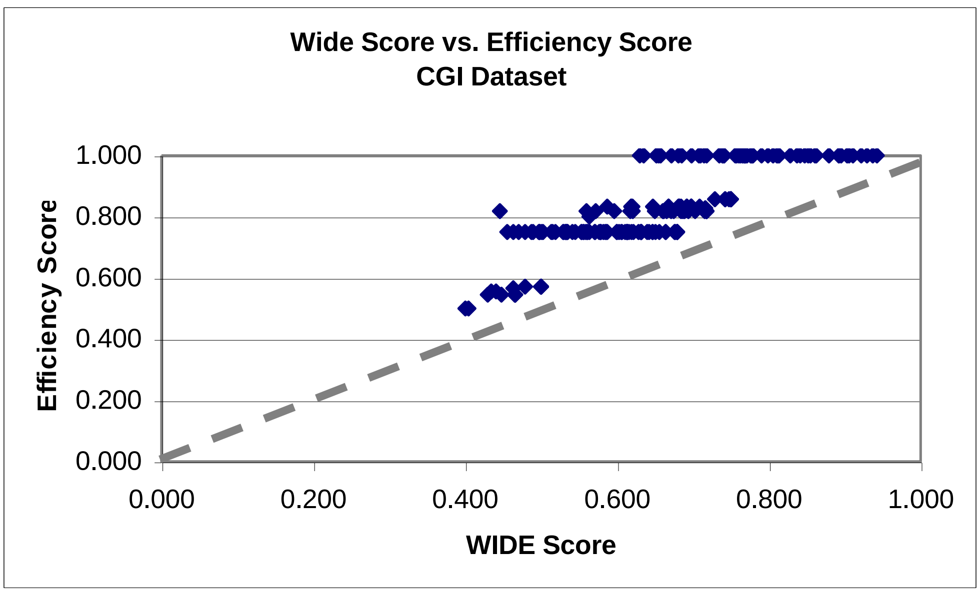


Figure 17 – Plot of Calculated WIDE score vs. Efficiency Score for OSD-CCR of GGA Dataset

Taking a closer look at Figure 17 we can see that using only the efficiency score that is provided by DEA (optimal objective function value from OSD-CCR formulation for each DMU) we obtain clusters of scores at different levels. For ranking purposes, the results are not very helpful because of the large number of ties at the 1.0 level and the 0.75 level. By plotting efficiency scores against WIDE scores we see an interesting effect of spreading out the DMUs that were previously tied and creating a definite rank order. This effect is witnessed not only for efficient units, but also for the inefficient units as well. The dashed line superimposed on Figure 17 corresponds to the line that the data points would follow if WIDE scores and efficiency scores were perfectly correlated. In this example, the results are well behaved in that no result touches or crosses the dashed line meaning that the results from the two approaches agree with each other but report different aspects. The additional information derived from the WIDE scores is an effective method to rank.

5.3.2 Discussion

It was found that the Efficiency score and WIDE scores from this analysis effectively classified DMUs as funded or not funded. The actual cutoff values for these scores to decide whether to fund or not fund were not determined as they would be data dependent and are only valid because we know what the actual outcome was. Furthermore, the actual outcomes of whether GGA funded a project cannot be taken as the gold standard and for many proposals, our results do not agree with the GGA. The GGA method is black and white while ours is a many-shaded colour of gray. The OSD/WIDE approach can be seen to be more robust than the current standard. Comparing the results between GGA and OSD shows that the top tier DMUs and the bottom tier DMUs are easy to identify for both. In fact, 100% of DMUs with a WIDE score in the top quartile were funded and 92% of the DMUs with a WIDE score in the bottom quartile were not funded. This is not surprising and validates both approaches. Contrasting that with results obtained using only the efficiency score we find that 90% of top quartile DMUs were funded and the bottom quartile of DMUs could not be identified due to the large concentration of DMUs

with a score of 0.75.

The WIF for this problem was [14%, 4%, 14%, 12%, 11%, 9%, 5%, 16%, 15%] allowing us to infer that the DMUs perceive Criteria 1 to be more than three as important as Criteria 2 and almost three times as important as Criteria 7. The direct comparison of perceived importance of all criteria to each other is one of the benefits of formulating MCDM problems with this methodology. If we knew the weights that the GGA uses for each of the criteria we could compare the two perspectives allowing the decision maker to ask questions like:

1. Why do the DMUs place so much importance on Criteria 8 and 9? Is this where they are more competitive?
2. If we wanted to be more fair to the DMUs should we consider changing our weighting of the criteria? If so we now have a target to move towards.
3. Why is Criteria 7 perceived to be so unimportant? Perhaps rethinking this criterion might provide more useful information or perhaps it could be excluded.

Overall, the results were promising from this analysis. We were able to calculate scores for each of the DMUs in a demonstrable fair way that mirrored the expert decisions without requiring expert opinion and, just as important, we are better able to defend the decisions made regarding the difficult middle group of alternatives.

5.4 CAPITAL CITY DATA RESULTS

For the purposes of illustrating the full RC methodology, across all three OSD models, a recent and important problem from the literature is treated here. Taken from [COOP00], we analyse the final weight selection and ranking for a site selection problem, previously analysed by Tone using a consensus-making method based on the assurance region (AR) method originally developed by Thompson et.al [THOM86] and enabled through the use of Saaty's AHP technique [SAAT80]. The goal being to present an alternative approach to the weighting of selection criteria and compare this weighting to expert opinion and ultimately, to compare the final ranking of alternative outcomes.

The OSD models are applied after all the criteria have been identified, the alternatives have been identified, and values for each of the criteria for each of the alternatives have been decided. At this point, the goal is to find weights to aggregate the criteria scores for ranking.

Alternative	C1	C2	C3	C4
A	5	10	3	5
B	7	10	3	10
C	8	7	10	5
D	4	8	3	10
E	9	4	4	2
F	10	5	10	3
G	4	7	7	8

Table 13 – Capital City Site Selection Raw Data

The site selection problem data is reproduced in Table 13 and shows 7 alternative sites that have been evaluated on 4 criteria.

Conveniently, all of the criteria are oriented in such a way that a higher value indicates a preferred site as required by OSD so data reversal is not necessary. Mapping all criteria to the full 0 to 1 scale can be done by simply dividing each value by 10 as presented in Table 14.

Alternative	C1	C2	C3	C4
A	0.5	1.0	0.3	0.5
B	0.7	1.0	0.3	1.0
C	0.8	0.7	1.0	0.5
D	0.4	0.8	0.3	1.0
E	0.9	0.4	0.4	0.2
F	1.0	0.5	1.0	0.3
G	0.4	0.7	0.7	0.8

Table 14 – Capital City Site Selection
Raw Data - Normalized

The assurance region model requires the specification of upper and lower bounds for criteria weights. In the example from the literature, this is accomplished by applying AHP techniques to elicit these bounds from five expert evaluators. Each of the evaluators were asked to distribute 10 *units* of weight across the four criteria. The results are reproduced in Table 15.

From here, the appropriate upper and lower bounds needed for the assurance region model are determined by examining the pair wise ratio of each weight to every other

weight for each evaluator.

Evaluator	W1	W2	W3	W4
1	1.67	3.33	1.67	3.33
2	2.11	3.16	1.58	3.16
3	2.50	1.88	1.88	3.75
4	2.00	2.00	2.00	4.00
5	2.40	1.90	1.90	3.80
Total	10.68	12.27	9.03	18.04
Normalized	**0.21**	**0.25**	**0.18**	**0.36**

Table 15 – Expert Evaluator Preferred Weights
for Capital City Site Selection

There will be 4!/2!*(4-2)! = 6 paired comparisons for the four criteria. The lowest value across the five evaluators for each is used as the lower bound and the highest value correspondingly is used for the upper bound as presented in Table 16.

Weight Ratio	Lower Bound	Upper Bound
W2/W1	0.75	2.00
W3/W1	0.74	1.00
W4/W1	1.50	2.00
W3/W2	0.50	1.00
W4/W2	1.00	2.00
W4/W3	2.00	2.00

Table 16 – Upper and Lower bounds
of weight ratios

At this point, the CCR assurance region model is run to calculate optimal weights for each alternative and one representative summary score which is then used to rank. The results, once again reproduced from the literature, are contained in Table 17 are found using the following LINDO program (all programs and raw results are available in Appendix C):

```
! CCR-AR Model for Capital City Site Selection

MAX 0.5 W1 + 1.0 W2 + 0.3 W3 + 0.5 W4

ST

0.5 W1 + 1.0 W2 + 0.3 W3 + 0.5 W4 <= 1
0.7 W1 + 1.0 W2 + 0.3 W3 + 1.0 W4 <= 1
0.8 W1 + 0.7 W2 + 1.0 W3 + 0.5 W4 <= 1
0.4 W1 + 0.8 W2 + 0.3 W3 + 1.0 W4 <= 1
0.9 W1 + 0.4 W2 + 0.4 W3 + 0.2 W4 <= 1
1.0 W1 + 0.5 W2 + 1.0 W3 + 0.3 W4 <= 1
0.4 W1 + 0.7 W2 + 0.7 W3 + 0.8 W4 <= 1

 0.75 W1 - 1.00 W2 <= 0
-2.00 W1 + 1.00 W2 <= 0
```

```
 0.74 W1 - 1.00 W3 <= 0
-1.00 W1 + 1.00 W3 <= 0

 1.50 W1 - 1.00 W4 <= 0
-2.00 W1 + 1.00 W4 <= 0

 0.50 W2 - 1.00 W3 <= 0
-1.00 W2 + 1.00 W3 <= 0

 1.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

 2.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

The addition of 12 constraints found in the CCR-AR model correspond to the upper and lower bounds found by eliciting expert opinion.

Alternative	Score	Rank	W1*	W2*	W3*	W4*
A	0.760	**6**	0.026	0.053	0.026	0.053
B	1.000	**1**	0.026	0.038	0.019	0.038
C	0.890	**2**	0.035	0.027	0.027	0.053
D	0.875	**3**	0.029	0.029	0.029	0.057
E	0.567	**7**	0.056	0.042	0.042	0.083
F	0.811	**5**	0.039	0.029	0.029	0.058
G	0.850	**4**	0.029	0.029	0.029	0.059

Table 17 – Final results for CCR-AR Site Selection Model

From here, Tone concludes that alternative site B is the best choice as it is the only one to achieve a score of 1.0 and thus ranks first. Given that expert opinion was used to set the upper and lower bounds and that each alternative was free to choose weights within those assurance regions that made them appear as attractive as possible, this seems like a valid choice.

The methodology presented above differs from the RC approach in that it is not centered around defining a consensus weighting of the criteria, but rather on deriving consensus on the scoring method and final ranking of the alternatives.

The RC methodology approaches the problem from a different perspective. It is concerned with defining weights for the selection criteria individually from each

alternative's perspective in such a way that the aggregation of those weights for scoring and ranking is acceptable to the alternatives themselves. In a site selection problem such as this, that may not be a method for making the final decision but it casts new light on the decision making process and allows for comparison of data derived weighting to the expert weighting already secured.

Next, we will analyse the same problem using three different OSD based approaches and then compare the weights found from each to expert weights and the final rankings found from each to those found using the CCR-AR model.

5.4.1 Summary of Results

OSD-CCR Analysis

The problem was first analysed using the original OSD-CCR formulation. It is convenient that the criteria values from the sample problem are all oriented in the correct manner where a higher value indicates a preferred choice. All results were obtained using the LINDO software package for linear and integer programming. All of the representative programs and LINDO output can be found in Appendix C while a sample for Alternative A is included inline for reference.

```
! OSD-CCR PROGRAM FOR ALTERNATIVE A

MAX 0.5 W1 + 1.0 W2 + 0.3 W3 + 0.5 W4

ST

0.5 W1 + 1.0 W2 + 0.3 W3 + 0.5 W4 <= 1
0.7 W1 + 1.0 W2 + 0.3 W3 + 1.0 W4 <= 1
0.8 W1 + 0.7 W2 + 1.0 W3 + 0.5 W4 <= 1
0.4 W1 + 0.8 W2 + 0.3 W3 + 1.0 W4 <= 1
0.9 W1 + 0.4 W2 + 0.4 W3 + 0.2 W4 <= 1
1.0 W1 + 0.5 W2 + 1.0 W3 + 0.3 W4 <= 1
0.4 W1 + 0.7 W2 + 0.7 W3 + 0.8 W4 <= 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

Using the formulation from Equation (4.1) and the program above, we obtain optimal weights for each alternative. Table 18 summarizes the results.

Evaluator	W1	W2	W3	W4
A	0	1	0	0
B	0	1	0	0
C	0	0.886	0.379	0
D	0	0	0	1
E	1	0	0	0
F	1	0	0	0
G	0.174	0	0.495	0.728
Total	2.174	2.886	0.875	1.728
Normalized	**0.283**	**0.376**	**0.114**	**0.225**

Table 18 – Optimal Weights found using OSD-CCR

The last row of the table corresponds to the WIF vector for this problem and can be interpreted as the consensus of the DMUs agreeing that 28%, 38%, 11% and 23% weight should be placed on criterion 1, 2, 3, and 4 respectively when calculating a score to rank. Table 19 presents the scores and ranks found using OSD-CCR.

Alternative	Score	Rank
A	0.665	**4**
B	0.835	**1**
C	0.718	**2**
D	0.675	**3**
E	0.497	**7**
F	0.654	**5**
G	0.637	**6**

Table 19 – Consensus Score and Rank found using OSD-CCR

OSD-IP Analysis

Evaluator	W1	W2	W3	W4
A	0	1	0	0
B	0	1	0	0
C	0	0	1	0
D	0	0	0	1
E	0.772	0	0.030	0.196
F	1	0	0	0
G	0	0	0.333	0.666
Total	1.772	2	1.363	1.863
Normalized	**0.253**	**0.285**	**0.194**	**0.266**

Table 20 – Optimal Weights found using OSD-IP

Moving next to the OSD-IP formulation which ranks alternatives based on aggregating calculated weights that maximizes the rank of each DMU, we find the following weights, scores and ranks summarized in Table 20 and Table 21. Note: In Table 20 we have renamed the first column to Evaluator from Alternative to reflect

the fact that each Alternative is now acting as an expert in determining its own weights for ranking.

Alternative	Score	Rank
A	0.604	**6**
B	0.788	**1**
C	0.731	**2**
D	0.655	**4**
E	0.473	**7**
F	0.671	**3**
G	0.651	**5**

Table 21 – Final Ranking found using OSD-IP

The representative LINDO program for Alternative A is:

```
!OSD-IP FOR ALTERNATIVE A

MIN Z1+Z2+Z3+Z4+Z5+Z6+Z7

ST

0.5 W1 + 1.0 W2 + 0.3 W3 + 0.5 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 - Z1 <= 0
0.7 W1 + 1.0 W2 + 0.3 W3 + 1.0 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 - Z2 <= 0
0.8 W1 + 0.7 W2 + 1.0 W3 + 0.5 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 - Z3 <= 0
0.4 W1 + 0.8 W2 + 0.3 W3 + 1.0 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 - Z4 <= 0
0.9 W1 + 0.4 W2 + 0.4 W3 + 0.2 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 - Z5 <= 0
1.0 W1 + 0.5 W2 + 1.0 W3 + 0.3 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 - Z6 <= 0
0.4 W1 + 0.7 W2 + 0.7 W3 + 0.8 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 - Z7 <= 0

0.5 W1 + 1.0 W2 + 0.3 W3 + 0.5 W4 = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END

integer Z1
integer Z2
integer Z3
integer Z4
integer Z5
integer Z6
integer Z7
```

The representative WIF vector for this problem based on the OSD-IP method is 25%, 29%, 20%, and 27% respectively.

OSD-DA Analysis

Using the following linear program, optimal weights can be found using the OSD-DA formulation. The Lindo program below sets the objective function to minimize the sum of all DMUs scores while constraining DMU A's score to 1.0.

```
! OSD-DA Formulation for DMU A
MIN 0.5 W1 + 1.0 W2 + 0.3 W3 + 0.5 W4 + 0.7 W1 + 1.0 W2 + 0.3 W3 + 1.0 W4
  + 0.8 W1 + 0.7 W2 + 1.0 W3 + 0.5 W4 + 0.4 W1 + 0.8 W2 + 0.3 W3 + 1.0 W4
  + 0.9 W1 + 0.4 W2 + 0.4 W3 + 0.2 W4 + 1.0 W1 + 0.5 W2 + 1.0 W3 + 0.3 W4
  + 0.4 W1 + 0.7 W2 + 0.7 W3 + 0.8 W4
ST
0.5 W1 + 1.0 W2 + 0.3 W3 + 0.5 W4 = 1
W1 > 0
W2 > 0
W3 > 0
W4 > 0
END
```

Evaluator	W1	W2	W3	W4
A	0	1	0	0
B	0	0	0	1
C	0	0	1	0
D	0	0	0	1
E	1	0	0	0
F	0	0	1	0
G	0	0	0	1
Total	1	1	2	3
Normalized	**0.142**	**0.142**	**0.285**	**0.428**

Table 22 – Optimal Weights found using OSD-DA

The optimal weights for each site can be found in Table 22.

Alternative	Score	Rank
A	0.514	**6**
B	0.757	**1**
C	0.714	**2**
D	0.686	**4**
E	0.386	**7**
F	0.629	**5**
G	0.700	**3**

Table 23 – Final Ranking found using OSD-DA

It is interesting to note that with the OSD-DA model, the optimal weights are always chosen by placing all weight on one criterion. The reason for this is that the program is designed to find a set of weights which distinguishes the DMU under investigation from the *average* DMU if it ranks above that average and to align itself to *catch up* with the average if it is below. Said another way, the program finds weights which

maximizes the positive distance of a DMU from average. In either case, there will always be one dimension on which the DMU under investigation can move along to maximize its movement towards this goal.

Taking the total of these weights and normalizing then gives a WIF vector for this model equal to 14%, 14%, 29% and 43%. The representative consensus weighting and resultant ranking based on OSD-DA is found in Table 23.

5.4.2 Discussion

The site selection problem has now been analysed using a number of complementary approaches such that we now have five sets of rankings and four sets of weights to compare and discuss. Table 24 provides a summary of the five different rankings of all seven alternative sites.

The second column of the table corresponds to the ranks that would have been assigned to each of the sites based solely on aggregating the expert weights and using those weights to score. The third column corresponds to the rankings found using the CCR assurance region model which incorporates those same expert weights. It is interesting to note that the overall ranking of all seven sites does not change when the CCR-AR model is incorporated. This could be interpreted to mean that expert ranking is aligned with DEA based ranking and that even with some freedom in choosing weights from the alternatives' perspective, the same overall ranking is maintained. However, this argument tends to break down when we look at the rankings derived using the OSD models as we can see many cases of rank reversals throughout the set.

Alternative	Rank Expert	Rank CCR-AR	Rank OSD-CCR	Rank OSD-IP	Rank OSD-DA
A	**6**	**6**	**4**	**6**	**6**
B	**1**	**1**	**1**	**1**	**1**
C	**2**	**2**	**2**	**2**	**2**
D	**3**	**3**	**3**	**4**	**4**
E	**7**	**7**	**7**	**7**	**7**
F	**5**	**5**	**5**	**3**	**5**
G	**4**	**4**	**6**	**5**	**3**

Table 24 – Comparison of final ranking from different approaches

All models chose Alternative B and C as ranking first and second respectively and Alternative E as ranking last. Ranks 3,4,5 and 6 however are not consistent among the three approaches. Most notably, when the extreme OSD-IP ranking tool is used, the third place rank is assigned Alternative F, which otherwise never ranks above position five.

	Consensus Weight				
Model	**W1**	**W2**	**W3**	**W4**	**Total**
Expert	**21**	**25**	**18**	**36**	**100**
OSD-CCR	**28**	**38**	**11**	**23**	**100**
OSD-IP	**25**	**29**	**19**	**27**	**100**
OSD-DA	**14**	**14**	**29**	**43**	**100**

Table 25 – Comparison of WIF Vectors from different approaches

One major benefit of the RC approach is being able to deal directly with ranks in a selection problem setting. Another major benefit is the ability to be able to compare weight distributions, encoded in the WIF vector, across different approaches and interpret this vector as different decision maker perspectives on the problem. Table 25 summarizes the four WIF vectors calculated for this problem and allows for the direct comparison of expert elicited weights to OSD calculated weights. The experts placed the most weight on Criterion 4 and the least amount of weight on Criterion 3. The OSD-CCR model placed the most weight on Criterion 2 and the least on Criterion 3. The OSD-IP model found that the most weight should be assigned to Criterion 2, agreeing with the OSD-CCR model but not to the same degree and also placed the least amount of weight on Criterion 3 agreeing with the experts and OSD-CCR. Overall, OSD-IP had the most balanced spread of weights across all criteria. The OSD-DA model placed almost half its weight on Criterion 4, while Criterion 1 and 2 shared the least amount of weight.

The objectives of each of these models in determining weights all lead to different weight vectors and different rankings of the alternatives. This is exactly the intended goal – not for the purpose of confusing the decision maker but for the purpose of providing more perspectives and hopefully engaging conversation and discussion on how weight selection translates to fairness and distinctness for example.

5.5 TWO-DIMENSIONAL ILLUSTRATIVE EXAMPLE

The following two-dimensional illustrative example was created to assist in the explanation and understanding of the mechanics of the models and methods used in this thesis. The ten data points were randomly chosen, coloured and labeled. The Lindo programs, results and summary sheets are located in Appendix D.

5.5.1 Step 1 – Collect & Orient Measures

The first step in performing an RC study is to identify the alternatives and collect and orient the measures on the multiple criteria being considered. For this illustrative example, we wanted to map the problem onto a 10x10 grid. Using an ad hoc approach, we created ten alternatives and randomly placed them on the grid.

Label	Alternative	Criteria 1	Criteria 2
A	Orange	1	8
B	Violet	2	5
C	Blue	3	8
D	Yellow	4	4
E	Cyan	4	2
F	Black	5	7
G	Red	6	5
H	White	7	7
I	Green	7	4
J	Grey	8	2

Table 26 – 2-D Example Raw Data

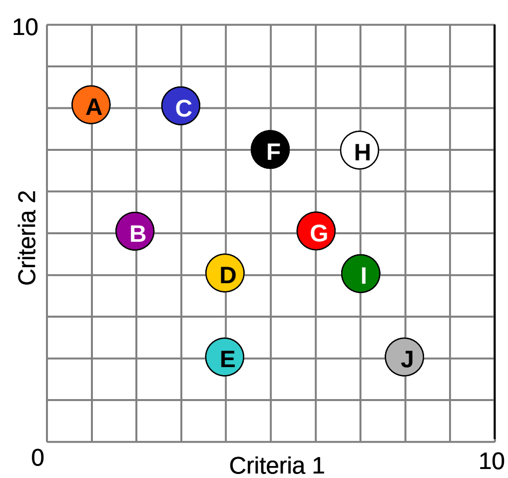


Figure 18 – Plot of 2D Simulated Raw Data

Table 26 presents the raw data used which is graphically presented in Figure 18.

The labels have been superimposed on the data points to aid in our discussion. We can assume that a better alternative is one that scores higher on any of the two criteria values so the measures are oriented appropriately.

5.5.2 Step 2 – Normalize Scales

The next step is to level the measurement scales for all criteria. This is accomplished by dividing all values measured for a criterion by the maximum possible value that could be achieved for that criterion. For our simple example, this is accomplished by dividing all values by ten, resulting in criteria values that are constrained to the full zero to one interval.

Label	Alternative	C1	C2
A	Orange	0.1	0.8
B	Violet	0.2	0.5
C	Blue	0.3	0.8
D	Yellow	0.4	0.4
E	Cyan	0.4	0.2
F	Black	0.5	0.7
G	Red	0.6	0.5
H	White	0.7	0.7
I	Green	0.7	0.4
J	Grey	0.8	0.2

Table 27 – 2-D Example Normalized Data

Table 27 and Figure 19 present and illustrate the problem after normalization.

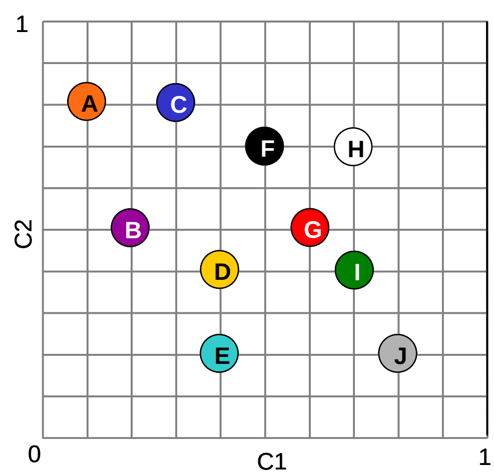


Figure 19 – Plot of 2D Simulated Data with normalized scales

5.5.3 Step 3 – Choose Weights

As with our previous applications and examples, we now have three new OSD models that can be used to calculate a weight vector for each alternative. For the purposes of demonstration, we will take each model in turn and illustrate graphically what each program is calculating.

OSD-CCR Model

$$\begin{aligned} \max \quad & w_o = \sum_{r=1}^{s} u_r y_{ro} \\ \text{st} \quad & \sum_{r=1}^{s} u_r y_{rj} \leq 1 \;\; j = 1,..,n \\ & u_r \geq 0 \end{aligned} \tag{5.1}$$

The OSD-CCR formulation is reproduced here for convenience where the *u* vector represents the optimal direction for an alternative under investigation and the *y* vector corresponds to the actual criteria values of the alternatives. Referring to Figure 20, we can now follow what the linear program is calculating when determining the optimal set of weights for alternative E (See Appendix D for illustrations of remaining alternatives).

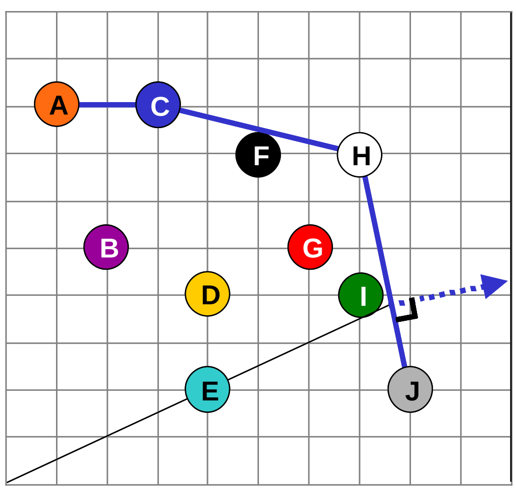


Figure 20 – Projection & optimal weights for Alt E using OSD-CCR

First, the program attempts to find a set of weights that will allow Alt E to 'rank first'. Mathematically, this is equivalent to drawing a hyperplane (represented as a

line in this two dimensional example) through Alt E such that no alternative would be 'above' (i.e. to the right or above) Alt E. In this example it is easy to see that Alt E is dominated in each direction by some other alternative, therefore, it cannot rank first – and thus is assigned a rank of second inline with current DEA practice. The weights that are assigned to Alt E are then calculated based on those alternatives that are able to rank first. In this example, for alternatives A,C,H and J, weight vectors do exist that allow them to rank first, therefore, these alternatives define the Pareto frontier. It is the orthogonal direction vector of the H-J facet of the frontier that is assigned to Alt E. This direction will minimize the distance between Alt E and the Pareto frontier without regard to overall rank. This is why we can say that the OSD-CCR model will 'choose weights to rank first, else rank second and assign weights to align with those alternatives that can rank first'.

Taking the radial projection of a vector from the origin through Alt E and touching the resultant frontier created by the linear combination of alternatives H and J gives a measure of distance which is used to calculate an efficiency score for Alt E. This is the score that the objective function of the OSD-CCR model is trying to maximize as illustrated in Figure 21 (See Appendix D for illustrations of remaining alternatives).

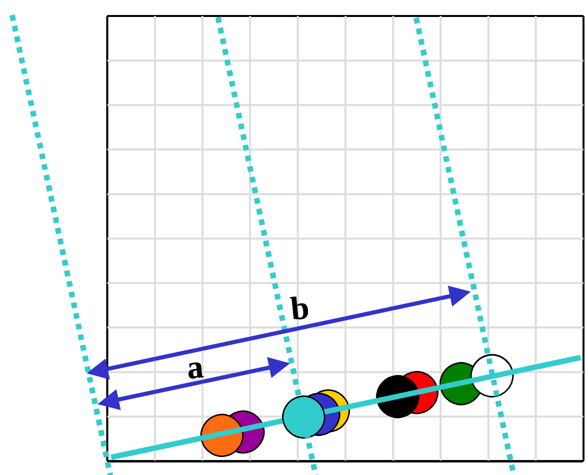


Figure 21 – Calculation of efficiency score using Alt E's OSD-CCR weights

The figure above illustrates what happens when we project all the alternatives to the weight vector calculated for Alt E. The ratio of 'a' over 'b' provides the resultant

score. The goal with the OSD-CCR model is to find those weights that maximizes the ratio of 'a' over 'b'. Looked at another way, it is attempting to maximize the value of 'a' relative to the value of 'b'. Choosing a different weight vector will result in different relative distances between the alternative under investigation and the reference alternative(s).

OSD-IP Model

The OSD-CCR model is centered on the concept of choosing weights to 'rank first'. This however, does not necessarily translate to the direction that will maximize an alternative's rank if it is not able to rank first overall. The OSD-IP formulation instead finds the set of weights that maximizes an alternative's rank.

$$\begin{aligned}
&\min \Omega = \sum_{j=1}^{n} z_j \\
&\text{st} \qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{s} u_r y_{ro} - z_j \le 0 \quad j = 1,\ldots,n \\
&\qquad\quad \sum_{r=1}^{s} u_r = 1 \\
&\qquad\quad u_r > 0 \\
&\qquad\quad z_j = 0 \; or \; 1
\end{aligned} \tag{5.2}$$

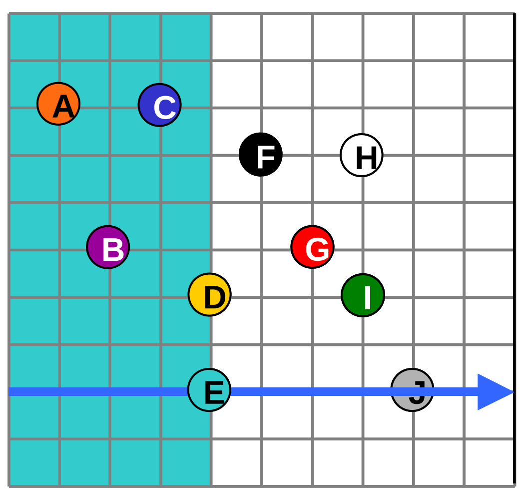


Figure 22 – Determination of Alt E's best weights using OSD-IP

This property is visible in this 2D example if we look at the same alternative, Alt E, and compare its best rank found using the OSD-CCR model and the OSD-IP model. While in the OSD-CCR model, the weights chosen minimizes Alt E's distance to the frontier (maximizes its relative score), there does exist another set of weights under which Alt E's rank is higher. Figure 22 illustrates this effect and Figure 23 shows what the ranking space looks like from Alt E's perspective.

If Alt E looks solely in the direction of Criteria 2 (along the X axis), then its max rank is 6, albeit tied for 6th. Any deviation from this direction will cause Alt D to move past Alt E in overall rank.

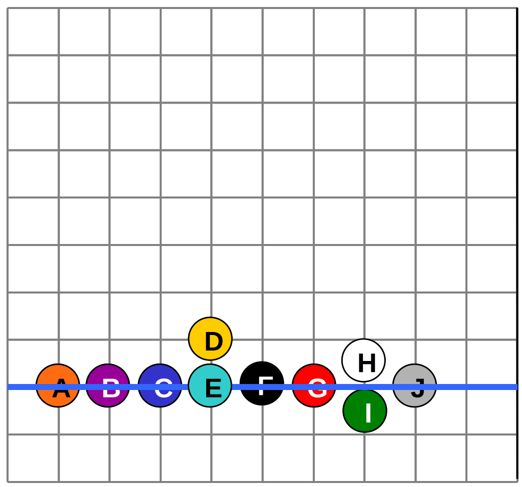


Figure 23 – Ranking from Alt E perspective using OSD-IP weights

For each alternative in this example, we can run the same program. Graphically, it has the effect of cutting the ranking space with a 1 dimensional hyperplane so as to minimize the number of alternatives above. Appendix D contains the Lindo programs and solutions for all alternatives.

OSD-DA Model

$$\begin{aligned} &\min \Phi = \sum_{j=1}^{n} \sum_{r=1}^{s} u_r y_{rj} \\ &\text{st.} \qquad \sum_{r=1}^{s} u_r y_{ro} = 1 \\ &\qquad\quad u_r > 0 \end{aligned} \tag{5.3}$$

Moving next to the OSD-DA model, we can now find the set of weights, which distinguishes the alternative under investigation from the average. The key to understanding what this program is doing graphically is to realize that the 'average' (marked by stars in Figure 24) changes depending on our perspective (weight vector chosen) and thus, the distance to this average will change as well.

Figure 24 displays this effect. As we scan across the entire ranking space, we find that Alt E's minimum distance from average is found using the weight vector [1,0] and represented by the blue one-sided arrow (See Appendix D for illustrations of remaining alternatives).

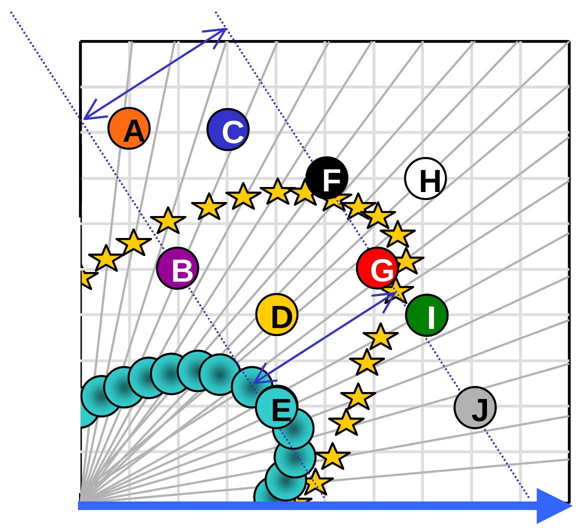


Figure 24 – Determination of Alt E's best weights using OSD-DA

The thin blue lines and double-sided arrows indicate the results when the weight vector [0.59,0.41] is arbitrarily chosen (equivalent to moving 10 units along the X axis and 7 units up the Y axis). At this point, the distance between the projection of Alt E and the average of all units' projections is greater than the optimal value.

It is interesting to note that for all weight choices, the score for Alt E is below the average. In general, this will not always be the case as in some directions an alternative will be above average, and in other directions it will score below average. The OSD-DA program automatically handles this property as it maximizes the positive distance of an alternative from the moving average. If an alternative is

below average it will find those weights that will minimize its distance, while if it is above average, it will maximize its distance.

Comparison of Chosen Weights from Three Models

Running all three models on all ten alternatives provides us with weight direction vectors illustrated in Figure 25. For comparison purposes, we have colour coded each weight direction vector to correspond to its respective alternative and displayed the results side by side to get a sense of the different weights that are found by using the three different approaches.

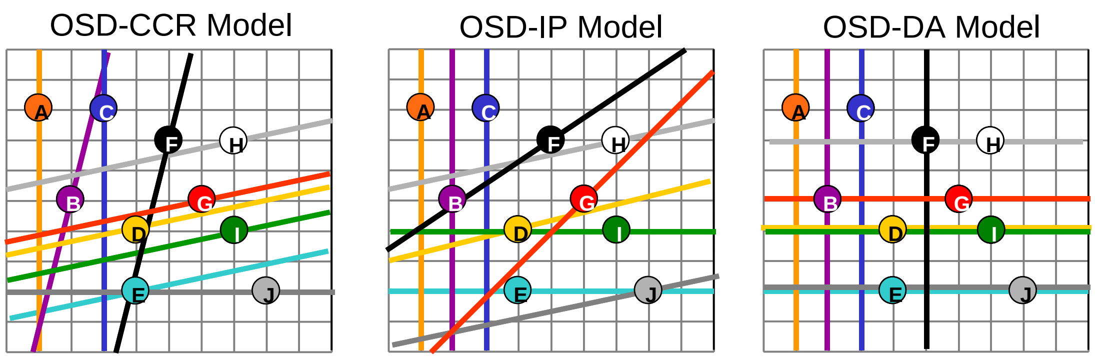


Figure 25 – Comparison of all alternatives' weights across three models

Focusing now on alternative G, we can see that using the OSD-CCR model, its weights are exactly aligned with unit E because they both radially project to the same facet defined by units H and J. This direction will maximize its relative score. However, if we move to the OSD-IP model, unit G now places more importance on Criteria 1, giving it a steeper direction vector and resulting in a move from 4th place to 3rd. Furthermore, changing focus from maximizing rank to distinguishing from average has the effect of unit G putting all its weight on Criteria 1, thereby completely flattening its weight vector.

In contrast, unit C, which is DEA efficient, always chooses the same weight vector to satisfy all three objectives. This is a function of its own criteria values combined with the relative values of all other alternatives in the space.

5.5.4 Step 4 – Stack Weights

The next step is to disconnect the weight direction vectors, normalize their magnitude (to allow for equal influence of all alternatives) and stack these weights for consensus.

Figure 26 illustrates the stacking of all ten individual weight vectors from the three different OSD models. The order in which we stack the weight vectors does not affect the resultant WIF calculated in the next step. Here, we have colour coded and stacked the vectors consistently to allow for comparison of the results from the three models.

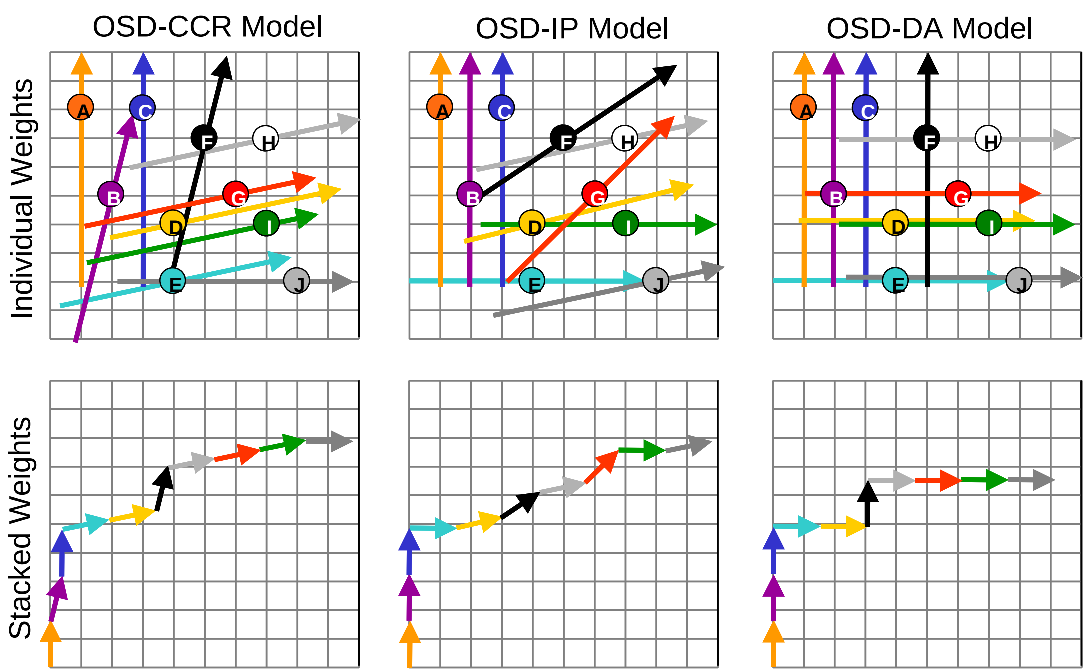


Figure 26 – Comparison of stacking of all weights across three models

The lower three graphs from Figure 26 present the normalized weight vectors for all ten alternatives. Of course, an analyst is free to adjust the magnitude of any or all weight vectors, inject their own weight vector or exclude others if necessary. Graphically, the length of a weight vector translates to the amount of influence that vector has in determining the resultant WIF vector.

5.5.5 Step 5 – Calculate WIF

Calculation of the WIF vector is found by taking the vector sum of all individual weight vectors. In Figure 27, the dashed blue arrow represents the WIF. It is interesting to note that while the OSD-CCR and OSD-IP models calculate individual weights with different intentions (and the resultant individual weights differ between the two models) coincidentally the resultant WIF is the same for both. The OSD-DA model results in a less steep WIF vector. In both cases, we are not concerned with the magnitude of the vectors at this stage, only the direction as the WIF is always normalized such that its components sum to 1.0.

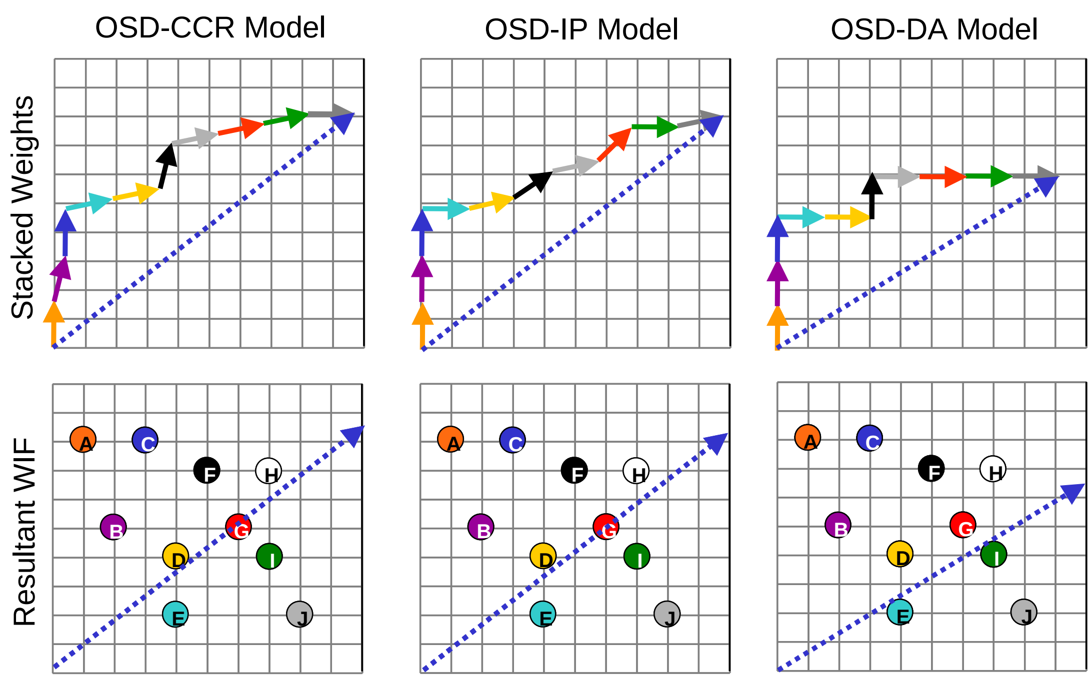


Figure 27 – Calculation and comparison of WIF vectors across three models

The second row of graphs from Figure 27 shows how the WIF vector is then mapped back onto the original ranking space to allow for Step 6 to be calculated.

5.5.6 Step 6 – Consensus Score

Step 6 is accomplished by taking the dot product of the normalized WIF vector with the original criteria values from Step 2.

Graphically, this calculation of score is equivalent to projecting all alternatives orthogonally onto the WIF vector. The scores of each alternative are now measures of distance from the origin to each of the alternatives. Dividing all scores by the max score (unit H in this example) will give relative scores between 0 and 1

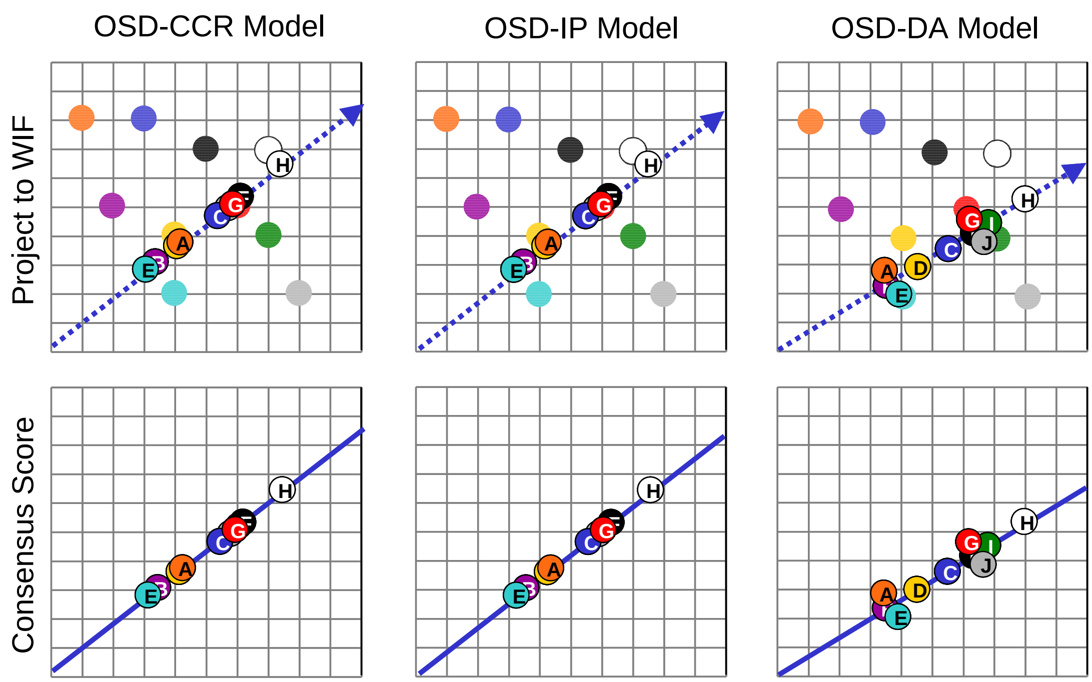


Figure 28 – Illustration of consensus scoring.

5.5.7 Step 7 – Consensus Rank

The previous step is concerned with determining the score of each alternative along the chosen consensus weight vector direction. Using those scores, we can now rank the alternatives. Graphically, ranking is the process of distributing the alternatives equidistantly along the WIF vector, while allowing for ties.

Using the weights found from the OSD-CCR and OSD-IP models, we find the final rankings of all alternatives to be 'clean' (i.e. no ties). This ranking was found without external expert opinion. Using the OSD-DA models to calculate our weights leads to a ranking formation with two ties, units G and J tied for 4th and units E and A tied for 9th. Once again these rankings were found without outside weights imposed and while unit H is ranked first using all three approaches, second and third

place rankings do change with units F and I trading places depending on our approach.

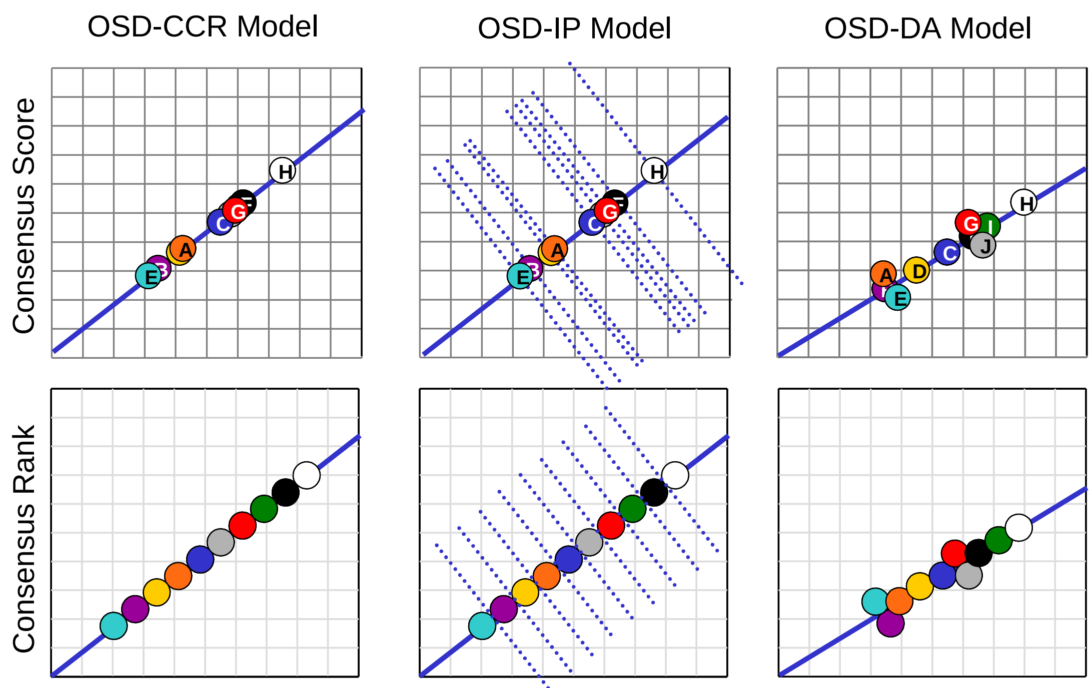


Figure 29 – Illustration of consensus ranking

5.5.8 Summary of 2D Results

Label	Alternative	OSD-CCR Ranking	OSD-IP Ranking	OSD-DA Ranking
A	Orange	7	7	8
B	Violet	9	9	9
C	Blue	6	6	6
D	Yellow	8	8	7
E	Cyan	10	10	9
F	Black	2	2	3
G	Red	4	4	4
H	White	1	1	1
I	Green	3	3	2
J	Grey	5	5	4

Table 28 – Summary of Rankings for 2D Example

The final rankings of all ten alternatives found using the three OSD models are presented in Table 28.

5.6 SUMMARY OF APPLICATIONS

The models and methods presented in Section 4.0 were tested in this Section on four different datasets. Table 29 summarizes the results. For each, the full RC methodology was performed (Steps 1 to Step 7) however different models and rationales were used for Steps 3 & 4 based on the data and additional information on the specific problems that were available to us.

Step 3 corresponds to the calculation of weights for each Alternative. We have developed three one-sided DEA based models for this purpose, each with a slightly different intention. The applications also differ in Step 4 (Stacking of Weights) in that not all DMUs necessarily have to contribute their weight choice to this step. In some situations one might want some DMUs to be excluded or conversely, some DMUs may be afforded a multiple of their influence.

In all cases we were able to calculate a meaningful WIF vector that was compared to expert opinion when available. Also, in each case, we found new rankings of the DMUs being analysed that were consistent and logical in the context of the applications being performed.

Data	No. Alt's	No. Criteria	Models Used	DMUs used for WIF	Compare WIF To	New Ranking	Comments
Power Plant	6	6	OSD-CCR	Efficient & Inefficient	N/A	Yes	4 Criteria were flipped
GGA	200	9	OSD-CCR	Only Inefficient	Expert Weights	Yes	
Capital City	7	4	OSD-CCR OSD-IP OSD-DA	All	Expert Weights	Yes	
2-D Simulated	10	2	OSD-CCR OSD-IP OSD-DA	All	N/A	N/A	Illustrates mechanics of models

Table 29 – Summary of Applications Performed

Ideally, each of the OSD models would be used and sensitivity analysis of the WIF vector to the inclusion or exclusion of efficient or inefficient DMUs would also be performed. This has been left to future work, as our purpose here was to demonstrate the methodology and illustrate its applicability across diverse problems.

6.0 CONCLUSIONS & FUTURE WORK

"The future belongs to those who believe in the beauty of their dreams." – Eleanor Roosevelt

To conclude this work, we will summarize the contributions made and explore future work and direction of the research. This thesis was focused on finding a way to rank alternatives when explicit expert opinion on the relative weight importance of criteria is not available. A new methodology was created that accomplishes this by deriving the consensus opinion of the alternatives as a proxy for the usual externally imposed decision maker preference. In essence, we created a self-ranking system. We were then able to demonstrate the approach on a number of relevant datasets, answering our original questions but also opening many new avenues for further research as we went along.

New applications and extensions continue to be discovered and developed, built on the core concepts and contributions put forth by this thesis. The remainder of this section will summarize those contributions and present a high level direction for further research.

6.1 SUMMARY OF CONTRIBUTIONS

There are three main contributions of this work, the creation of the RC Methodology, development of the spectrum of OSD models and application of such methodologies to four relevant datasets.

6.1.1 New RC Methodology

Seven Steps of RCM

A new Ranking Methodology was created based on a simple seven-step process and standardized ranking space. By orienting all criteria measures in a positive manner (Step 1), and normalizing measures to the full zero to one scale (Step 2), we enabled a way to map alternatives in a ranking problem into a standardized n-dimensional

vector space. This then lead to the creation of OSD models to calculate preference weights for each alternative (Step 3). Aggregating those weights (Step 4) allows us to infer consensus (Step 5), which is then used to score (Step 6) and rank (Step 7) the alternatives.

WIG's, WIFs and WIDE Scores

Weight Importance Graphs (WIGs), Weight Importance Factors (WIFs) and Weight Derived Efficiency Scores (WIDE Scores) were all created and defined for this work to allow for the interpretation and analysis of the RC methodology.

The weights calculated by the OSD models can be sorted in decreasing order and plotted for each criterion creating a WIG whose shape and area represent the consensus of DMUs implied importance of that criterion. This visualization allows management to quickly assess a ranking problem in terms of trade-offs between criteria and also interpret perceived importance of criteria from a whole new perspective.

The area under each curve is normalized with all others to create a vector of weights (WIF) that gives an overall picture of the implied importance of all criteria. The resultant WIF vector provides managers and experts with a common language of evaluation for comparing their perspective with others. We extended the basic concept of the weight importance factor to the case where alternatives can have varying influence on the resultant vector and abstracted it again so that Experts can impose their perspective and manually adjust the relative weighting of the perspective of the alternatives with their own. The calculated WIF or expert adjusted WIF can then be applied to each alternative's criteria values to produce a new WIDE score. The WIDE score is a useful management tool in that it distills a multi-dimensional problem down to a single summary index for ranking.

Comparison of OSD calculated weights to MCDM elicited weights

The creation of the OSD models helps bridge the gap between traditional MCDM and DEA theory which was a stated goal of this research. MCDM is focused on

expert elicited weights while DEA's primary advantage for our purposes is its independence from expert input. Our models take the benefits of DEA and the methodology of deriving weights using standard linear programming techniques and format the results such that those weights can be compared to expert derived weights if available, or analysed on their own. The significant difference in thinking that these models embody is the attachment of the meaning of *Weight Importance* to DEA derived weights. Being able to infer the implied Weight Importance of each criterion from the DMUs perspective is especially critical when the methodology for selecting winning DMUs out of a population must be defended to the DMUs themselves.

6.1.2 New OSD Models

In order to investigate the weights assigned to each criterion in using DEA approaches, a spectrum of new One-Sided DEA models were created. These models move all criteria that a DMU is being evaluated against onto one side of the traditional DEA model. The resultant weights calculated can then be compared to Expert elicited weights in a traditional MCDM sense and they can also be investigated and analyzed on their own to gain new insight into selection problems and the selection process.

These new models avoid the pitfalls identified in the literature with using DEA on MCDM problems.

OSD-CCR Formulation

A simplification of the standard CCR model from the DEA literature was derived. The OSD-CCR formulation is equivalent to the original CCR formulation when all variables in the model are considered to be outputs and one single constant input is used. Equivalence was demonstrated by deriving the new formulation from the original CCR model. Equivalence can also be seen through inspection in that the OSD-CCR formulation will maximize a DMUs score while constraining every DMU's score to be less than or equal to 1.0, which is the essence of CCR DEA.

OSD-IP Formulation

The creation of the OSD-IP formulation calculates a set of weights for each DMU that maximizes its rank. This property allows us to calculate a common set of weights to evaluate and rank alternatives in a fairer manner in a selection problem setting. The benefit of this new DEA formulation is that it reduces push back from the alternatives on the set of weights used for ranking because each DMU can contribute an equal vote in the consensus weight and that vote is based on the optimal distribution of weights from its perspective, with full knowledge of all other DMUs criteria scores.

OSD-DA Formulation

Building on the concepts from the basic OSD models and extending from the OSD-IP formulation, a new OSD-DA formulation was created and applied which derives weights for each DMU with a clear objective of distinguishing them from the average. In today's competitive business environment, being able to identify those DMUs which stand out from the crowd and are able to differentiate themselves is an attractive proposition. Also, when defining a common set of weights to apply to all DMUs for planning or policy decisions, this model appears the most appropriate.

OSD Extensions

Another contribution of the OSD methodology is the concept of being able to introduce *direction* into a ranking study. The OSD extensions derived as part of this work are only a sample of those that could be created to exploit this new quality of the model.

6.1.3 Application to Relevant Datasets

The RC methodology was tested on four relevant datasets, 1) Power Plant Site Selection, 2) Research Grant Proposal Screening 3) Capital City Site Selection and 4) Two-dimensional simulated data for illustration. Each application showed new insight into the decision process and provided new rankings of the alternatives.

Overall, the results from each of the applications showed consistent ranking of the top performers as well as the bottom performers. It was the difficult middle group that demonstrated the most variability in rank position when different weight schemes were applied. This is further evidence that the models and methods are robust in that the final rankings reflect the intention behind the weight calculation without being overly sensitive to manipulation (i.e. those that dominate tend to dominate across scenarios).

6.2 FUTURE WORK

This research has shown that there exists a significant opportunity to further develop the theory surrounding the use of DEA as a tool for problems amenable to MCDM solutions.

The work done to date lays the foundation for analysing these types of problems in a new way. With the developed RC Methodology, OSD models and related WIGs, WIFs and WIDE Scores, we now have the beginnings of a comprehensive toolbox for ranking alternatives and for comparing DEA derived weights (with or without expert opinion) to MCDM Expert derived weights. Further investigation into why DMUs place more importance on certain criteria and what that means in terms of competitiveness, performance and goal setting among the DMUs on those criteria is still to be performed.

Also, presenting a methodology for incorporating weight bounds in the new OSD model will need to be completed. Preliminary results show that the models and approach introduced here will make the inclusion of expert opinion more intuitive than the assurance region and cone ratio techniques that are presently the standard.

Sensitivity analysis of the WIF vector to which units are included in its determination needs to be performed. A method for determining the sensitivity of individual DMUs to changes in the WIF is another area that appears relevant for further study. Specifically, looking at how swings of importance between different criteria and which direction that translates into in terms of moving up or down ranks

would provide further insight into the area of connecting WIFs with competitiveness amongst alternatives.

One area that appears interesting is to investigate the *shape* of the WIGs that are created in an RCM study. This idea becomes more interesting as the number of alternatives evaluated in a study increases.

A multi-stage approach to weight determination inline with the aggressive and benevolent cross-efficiency formulations needs to be investigated and developed. It is hypothesized that this can be accomplished by combining OSD models and/or applying goal programming techniques.

Development of a method to determine a threshold of distinguishability between ranks needs to be performed, tested and rigorously specified. Also, extending the research to use more sophisticated decision models should be a high priority.

Lastly, the application of RCM to other datasets should be examined to test the predictive capability of the model and derive new insight into the scoring, ranking and selection of alternatives. Work in this area will contribute to a more formalized framework for which models are most appropriate in certain contexts which remains one of the main limitations of our approach.

It is hoped that this work will be an impetus to more formally bring together somewhat disparate areas of management science, including Performance Assessment, Multi Criteria Decision Making and Data Envelopment Analysis for the common good and benefit of each. It is also the sincere hope of the author that the simplicity, scalability and interpretability of these new ranking tools will lead to their timely adoption and application.

7.0 EPILOGUE

"If a man is considered guilty for what goes on in his mind,
give me the electric chair for all my future crimes." – Prince

I am a Computer Engineering & Management graduate from McMaster University with an MBA from the Michael G. DeGroote School of Business. My passion is technology commercialization. A technology is simply 'a way of doing something'. Commercialization is simply 'exploiting for profit'. I have studied and worked with many new technologies attempting to understand the process of going from that first flash of insight to cash in hand. I learned the hard way that success in this area depends mostly on three keys issues:

1. Clarity of ownership
2. Ease of demonstration
3. Clarity of liquidity

This thesis presents a new technology for ranking. What is the ultimate application of a new ranking technology in the context of commercialization? There are many good applications; credit scoring, debt management, rank ordering insurance claims, pricing systems, customer relations management, to name but a few. Many socially good applications exist including triage, disaster recovery, strategic planning and policymaking. There are also many fun applications like team and player ranking, Olympic Medal ranking, and matching as examples. Each of these applications holds promise but their successful commercialization depends on the three key issues. Ownership is clear so that is not a problem. Demonstration is the major bottleneck for most of the applications. Clarity of liquidity refers to how easy it is to see how wealth will be created by application of the technology. While troublesome for some applications, creating a valid business case for each is not overly difficult.

This does not answer the original question though. What is the ultimate application of a new ranking technology in the context of commercialization? It has to be easy to demonstrate and the connection to wealth creation has to be almost transparent. It

has to be something that people will actually use and want to use. I was not looking for the answer to this question when it came to me a little while back. I was looking for a way to explain the mechanics of the mathematics developed in this thesis to a non-technical person when the ultimate application (from the perspective of technology commercialization and specifically the three key issues) fell into my lap.

The experience was exactly the proverbial light bulb going on as I gathered a deck of ordinary playing cards and a handful of change from my living room to play the first casino game based on this new technology. From there, a whole suite of games can be created based on the same core idea – extending the concept into many exciting application areas – even board games. Sounds unbelievable does it not?

My personal interest is to follow through and carry this technology through the remainder of the commercialization process. Patents, publishing, and licensing are the next key steps as I understand it. The technology developed in this thesis forms a perfect case study for learning and understanding the process as a whole. My desire is to see it through and to see the full potential of this breakthrough realized, for the good of all.

REFERENCES

[ANDE93] Andersen, P., Petersen, N.C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science*, **39**(10), 1261-1294.

[BAKE97] Baker, R. C. and S. Talluri (1997). A closer look at the use of Data Envelopment Analysis for technology selection. *Computers Industrial Engineering*, **32**(1), 101-104.

[BANK84] Banker, R.D., Charnes, A., & Cooper, W.W., (1984). Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis. *Management Science*, **30**(9), 1078-1092.

[BANK86a] Banker, R.D., Morey, R., Efficiency Analysis for Exogenously Fixed Inputs and Outputs (1986), *Operations Research*, **34**(4),513-521.

[BANK86b] Banker, R.D., Morey, R., The Use of Categorical Variables in Data Envelopment Analysis, (1986), *Management Science* **32**(12), 1613-1627.

[BUCH01] Buchanan, J., Kock, N. (2001). Information Overload: A Decision Making Perspective, In: Koksalan, M. and S. Ziomts (eds.). Multiple Criteria Decision Making in the New Millennium, Lecture Notes in Economics and Mathematical Systems, **507**, 49-58, Berlin: Springer-Verlag.

[CHAR78] Charnes, A., Cooper, W.W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research,* **2**(6), 429-444.

[CHAR82] Charnes, A., Cooper, W.W., Seiford, L.(1982). A Multiplicative Model for Efficiency Analysis, Socie. Econ. Plan. Sci., **16**(5), 223-224.

[CHAR89] Charnes, A., Cooper, W.W., Wei, Q.L. & Huang, Z.M. (1989). Cone-ratio Data Envelopment Analysis and multi-objective programming. *Int. J. System. Sci*., **20**, 1099-1118.

[CHAR97] Charnes, A., Cooper, W., Lewin, A.Y., Seiford, L.W. (1997). Data Envelopment Analysis: Theory, Methodology and Application, Kluwer Academic Publishers.

[CHEH98] Chehade, R., (1998). Mutual fund performance evaluation using DEA. M.A.Sc. Thesis, Department of Mechanical & Industrial Engineering, University of Toronto, Canada.

[COOP96] Cooper, W.W., Thompson, R.G., & Thrall, R.M., (1996). Extensions and new developments in DEA, Annals of Operations Research, **66**, 9.

[COOP00] Cooper, W.W., and Seiford, L.M., Tone, K. (2000) Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References, and DEA-Solver Software

[DESP02] Despic, O., (2002). Linguistic Analysis of Efficiency using Fuzzy Systems Theory and Data Envelopment Analysis, Ph.D. Thesis, CMTE, University of Toronto.

[DOYL93] Doyle, J. and R. Green (1993). Data Envelopment Analysis and Multiple Criteria Decision Making, *Omega International Journal of Management Science*, **21**(6), 713-715.

[DOYL94] Doyle, J. and R. Green (1994). Efficiency and cross-efficiency in DEA: derivations, meanings and uses. *Journal of the Operational Research Society,* **45**(5), 567-578.

[DOYL95A] Doyle, J. R. (1995). Multiattribute choice for the lazy decision-maker - let the alternatives decide. *Organizational Behavior And Human Decision Processes,* **62**(1), 87-100.

[DOYL95B] Doyle, J. R. and R. H. Green (1995). Cross-evaluation in DEA - improving discrimination among dmu's. *INFOR*, **33**(3), 205-222.

[FARR57] Farrell, M.J., (1957). The Measurement of productive efficiency. *Journal of the Royal Statistical Society*, **120**(3), 253-290.

[FRIE97] Friedman, L., Sinuany-Stern, Z., (1997). Scaling units via the canonical correlation analysis and the data envelopment anlaysis. *European Journal of Operational Research,* **100**(3), 629-637.

[GATT04A] Gattoufi, S., Oral, M., Kumar, A., Reisman, A. (2004). Epistemology of data envelopment analysis and comparison with other fields of OR/MS for relevance to applications, *Socio-Economic Planning Sciences*, **38**, 123-140.

[GATT04B] Gattoufi, S., Oral, A., Reisman, A. (2004). A taxonomy for data envelopment analysis, *Socio-Economic Planning Sciences*, **38**, 141-158.

[GATT04C] Gattoufi, S., Oral, M., Reisman, A. (2004). Data envelopment analysis literature: a bibliography update (1951-2001), *Socio-Economic Planning Sciences*, **38**, 159-229.

[GELD02] Geldermann, J., Zhang, K., Rentz, O. (2002). Multi-criteria group decision support for integrated technique assessment. French-German Institute for Environmental Research Working Paper, University of Karlsruhe, Germany.

[GREE87] Greenberg, R., and Nunamaker, T. (1987). A generalized multiple criteria model for control and evaluation of nonprofit organizations, *Financial Accountability & Management*, **3**(4), 331-343.

[GREE95] Green, R.H. and J. Doyle (1995). On maximizing discrimination in multiple criteria decision making. *Journal of the Operational Research Society*, **46**, 192-204.

[HANS02] Hansen, Paul (2002). Another Graphical Proof of Arrow's Impossibility Theorem, *Journal of Economic Education*, Summer, **33**(3), 217-235.

[HIBI94] Hibiki, N. and T. Fukukawa (1994). Modified Cross-Efficiency in DEA and Its Application, Department of Administration Engineering, Keio University.

[HOSS04] Hosseini Ardehali, P. (2004). Assessing financial risk tolerance of portfolio investors using data envelopment analysis. M.A.Sc. Thesis, CMTE, University of Toronto.

[HWAN81] Hwang, C.L., and Yoon, K. (1981), Multiple Attribute Decision Making: Methods and Applications, A State-of-the-Art Survey. Berlin: Springer Verlag.

[JESS01] Jessop, Alan (2001). Multiple Attribute Probabilistic Assessment of the Performance of Some Airlines. In: Koksalan, M. and S. Ziomts (eds.). Multiple Criteria Decision Making in the New Millennium, Lecture Notes in Economics and Mathematical Systems, **507**, 417-425, Berlin: Springer-Verlag.

[KEEN76] Keeney, R., and Raiffa, H. (1976). Decisions with Multiple Objectives: preferences and Value Trade-offs. John Wiley & sons, New York.

[KHOO02] Khoo, P. (2002). The Corrected Frontier – DEA Chebyshev model with an application to financial resource management. Ph.D. Thesis, CMTE, University of Toronto.

[KHOU95] Khouja, M. (1995), The use of Data Envelopment Analysis for technology selection. *Computers Ind. Engng.*, **28**,123-132.

[MAIS98] Maisel, I. (1998). The puzzle comes together, *Sports Illustrated.* New York: **88**(16), pg 73.

[MERR02] Merritt, J. (2002). The best b-schools Kellogg vaults to No. 1. *Business Week*. New York, Iss. 3804, pg 84.

[MORR00] Morris, Dick.(2000). The New Prince: Machiavellian Updated for the Twenty-First Century, New York: Renaissance Books.

[OLSO01] Olson D., Mechitov A., and Moshkovich H. (2001). Learning Aspects of Decision Aids. In: Koksalan, M. and S. Ziomts (eds.). Multiple Criteria Decision Making in the New Millennium, Lecture Notes in Economics and Mathematical Systems, **507**, 41-49, Berlin: Springer-Verlag.

[ORAL91] Oral, M., O. Kettani, et al. (1991). A Methodology for collective evaluation and selection of industrial R&D projects. *Management Science,* **37**(7), 871-885.

[PARA04] Paradi, J.C., et.al (2004). Ranking by Consensus Methodology. Working Paper, Centre for Management of Technology & Entrepreneurship, University of Toronto.

[ROLL91] Roll Y, Cook W and Golany B (1991). Controlling factor weights in Data Envelopment Analysis. *IIE Transactions,* **23**(1), 2-9.

[ROUA03] Rouatt, S. (2003). Two stage evaluation of bank branch efficiency using data envelopment analysis. M.A.Sc. Thesis, CMTE, University of Toronto.

[ROY90] Roy, B. (1990). The outranking approach and the foundation of ELECTRE methods. In Carlos Bana E Costa, editor, *Readings in multiple criteria decision aid*, Springer, Heidelberg, 155-183.

[ROY96] Roy, B. (1996). Multicriteria methodology for decision aiding. Kluwer Academic Publishers, Dordrecht.

[SAAT80] Saaty, T.L. (1980). The Analytic Hierarchy Process. McGraw-Hill, NewYork.

[SEXT86] Sexton, T. R., (1986). The Methodology of Data Envelopment Analysis. R.H. Silkman (Ed.) *Measuring Efficiency: An Assessment of Data Envelopment Analysis,* San Francisco, Jossey-Bass.

[SIMA99] Simak, P., (1999), Inverse and Negative DEA. Ph.D. Thesis, Department of Mechanical & Industrial Engineering, University of Toronto, Canada.

[SINU98] Sinuany-Stern, Z., Friedman, L., (1998). Data envelopment analysis and the discriminant analysis of ratios for ranking units. *European Journal of Operational Research*, **111**, 470-478.

[SOWL04] Sowlati, T., Paradi, J.C. (2004). Establishing the practical frontier in data envelopment analysis. *OMEGA International Journal of Management Science*, **32**(4), 261-272.

[STEW92] Stewart, T.J. (1992). A critical survey on the status of multiple criteria decision making theory and practice. *Omega*, **20**, 569-586

[STEW94] Stewart, T.J. (1994). Data envelopment analysis and multiple criteria decision making: a response. *OMEGA International Journal of Management Science*, **22**(2), 205-206.

[THAN96] Thanassoulis, E., Boussofiane, A., Dyson, R.G. (1996). A comparison of data envelopment analysis and ratio analysis as tools for performance assessment, *Omega*, **24**(3), 229-244.

[THOM90] Thompson, RG, Langemeier, LN, Lee, CT and Thrall RM (1990). The Role of multiplier bounds in efficiency analysis with application to Kansas farming. *Journal of Econometrics*, **46,** 93-108.

[TOFA96] Tofallis, C. (1996), Improving Discernment in DEA Using Profiling, *Omega, International Journal of Management Science,* **24**(3), 361-364.

[TONE01] Tone, K. (2001). A slacks-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research*, **130**, 498-509.

[TRIA00] Triantaphyllou, E. (2000). Multi-criteria decision making methods: a comparative study. Kluwer Academic Publishers.

[YOON95] Yoon, K., and Hwang, C. L. (1995), Multiple Attribute Decision Making: An Introduction. *Thousand Oaks*, *California*, *Sage.*

[ZELE82] Zeleny, M., (1982), Multiple Criteria Decision Making. New York, McGraw-Hill.

BIBLIOGRAPHY

[ATHA97] Athanassopoulos, A.D. (1997). Dominance and potential optimality in multiple criteria decision analysis with imprecise information, *The Journal of the Operational Research Society*, February 1997.

[BOUY99] Bouyssou, D. (1999). Using DEA as a tool for MCDM: some remarks, *Journal of the Operational Research Society*, **50**, 974-978.

[CHAR85] Charnes, A., Cooper, W.W., Golany, B., & Seiford, L. (1985), Foundations of Data Envelopment Analysis for Pareto-Koopmans efficient Empirical Production Functions. *Journal of Econometrics*, **30**, 91-107.

[CHAR94] Charnes, A., Cooper, W.W., Lewin, A.Y., & Seiford, L.M.(1994),Data Envelopment Analysis, Theory, Methodology and Applications. *Kluwer Academic Publishers.* Boston.

[COOK96] Cook, W.D. et al. (1996). Data Envelopment Analysis in the Presence of Both Quantitative and Qualitative Factors, *Journal of the Operational research Society*, **47**, 945-953.

[COOP93] Cooper, R. G. (1993). Winning at New Products: Accelerating the Process from Idea to Launch, Addison-Wesley Publishing Company, Inc.

[DOYL91] Doyle, J. and R. Green (1991). Comparing products using data envelopment analysis, *Omega*, **19**, 631-638.

[GOLA89] Golany, B. and Y. Roll (1989). An Application Procedure for DEA, *OMEGA*, **17**(3), 237-250.

[GREE96] Green, R.H., Doyle, J.R., & Cook, W.D. (1996). Preference voting and project ranking using DEA and cross-evaluation. *European Journal of Operational Research*, **90**, 461-472.

[KAUF00] Kauffman, P., Unal, R., et. al.(2000). A model for allocating resources to research programs by evaluating technical importance and research productivity, *Engineering Management Journal*, **12**(1), 5-8.

[KORH01] Korhonen, P., Tainio, R. and Wallenius, J. (1999). Value efficiency analysis of academic research. *European Journal of Operational Research*, **130**, 121-132.

[LI99] Li, X. and G.R. Reeves (1999). A multiple criteria approach to data envelopment analysis, *European Journal of Operational Research*, **115**, 507-517.

[LOVE95] Lovell, K.C.A. and J.T. Pastor (1995). Units invariant and translation invariant DEA models, *Operations Research Letters*, **18**, 147-151.

[PARK97] Parkan, C. and M. Wu (1997). On the equivalence of operational performance measurement and multiple attribute decision making. *International Journal of Production Research*, **35**(11), 2963-2988.

[PARK99] Parkan, C., Wu, M. (1999). Decision-making and performance measurement models with applications to robot selection. *Computers & Industrial Engineering*, **36**, 503-523.

[PARK00] Parkan, C. and M. Wu (2000), Comparison of three modern multicriteria decision making tools, *Interanltional Journal of Systems Science,* **31**(4), 497-517.

[ROLL93] Roll, Y. and Golany, B. (1993). Alternate methods of treating factor weights in DEA. *OMEGA*, **21**(1), 99-109.

[SARK97] Sarkis, J. (1997). Evaluating Flexible Manufacturing Systems Alternatives Using Data Envelopment Analysis, *The Engineering Economist*, **43**(1), 25-47.

[SARR97] Sarrico, C.S., Hogan, S.M., et. al. (1997). Data envelopment analysis and university selection, *Journal of the Operational Research Society*, **48**, 1163-1177.

[SARK00] Sarkis, J. (2000). A comparative analysis of DEA as a discrete alternative multiple criteria decision tool, *European Journal of Operational Research*, **123**, 543-557.

[SEIF90] Seiford, L.M. and A. Iqbal Ali (1990). Translation invariance in data envelopment analysis, *Operations Research Letters – North Holland*, **9**, 403-405.

[SEIF96] Seiford, L.M., (1996). Data Envelopment Analysis: The Evolution of the State-of-the-Art (1978--1995), *Journal of Productivity Analysis,* **7**(2/3) 99-137.

[SEIF98] Seiford, L.M. (1998) An acceptance system decision rule with data envelopment analysis. *Computers & Operations Research*, April 1998.

[SHAN95] Shang, J. and T. Sueyoshi (1995), A unified framework for the selection of a Flexible manufacturing System, *European Journal of Operational Research*, **85**, 297-315

[STEW96] Stewart, T.J. (1996). Relationships between data envelopment analysis and multicriteria decision analysis, *Journal of the Operational Research Society,* **47**, 654-665.

[TALL97] Talluri, S., and J. Sarkis (1997). Extensions in Efficiency measurement of alternate machine component grouping solutions via data envelopment analysis. *IEEE Transactions on engineering Management*, **44**(3) 299-304.

[THAN98] Thanassoulis, E., Allen, R. (1998). Simulating weights restrictions in data envelopment analysis, by means of unobserved dmus, *Management Science*, **44**(4), 586-594.

[WEI00] Wei, Q. et. al. (2000). An inverse DEA model for inputs/outputs estimate, *European Journal of Operational Research*, **121**, 151-163.

[WONG90] Wong, Y.-H., and Beasley, J.E. (1990), Restricting weight flexibility in data envelopment analysis, *Journal of the Operational Research society,* **41**, 829-835.

[YOON80] Yoon, K. (1980) Systems selection by multiple attribute decision making. PhD Thesis, Kansas State University, Manhattan, Kansas.

APPENDIX A: POWER PLANT SITE SELECTION LINDO PROGRAMS, OUTPUT & SUMMARY SHEETS

Evaluation of Candidate Sites for Power Plant [STEW92].

Raw Data:

Location	(I) Manpower	(O) PowGen	(I) ConstructCost	(I) MaintenCost	(I) VillagesEvac	(O) SafetyLevel
Italy	80	90	600	54	8	5
Belgium	65	58	200	97	1	1
Germany	83	60	400	72	4	7
UK	40	80	1000	75	7	10
Portugal	52	72	600	20	3	8
France	94	96	700	36	5	6

Lindo programs and output for OSD-CCR model

Italy – OSD-CCR

```
! Italy OSD-CCR STEP3

max 0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6

st

0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6<=1
0.615384615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6<=1
0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6<=1
1W1+0.833333333333333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6<=1
0.769230769230769W1+0.75W2+0.3333333333333W3+1W4+0.3333333333333W5+0.8W6<=1
0.425531914893617W1+1W2+0.285714285714286W3+0.5555555555555W4+0.2W5+0.6W6<=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end
```

Belgium – OSD-CCR

```
! Belgium OSD-CCR STEP3
```

```
max
0.615384615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6

st

0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6<=1
0.615384615384615W1+0.60416666666667W2+1W3+0.206185567010309W4+1W5+0.1W6<=1
0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6<=1
1W1+0.833333333333333W2+0.2W3+0.26666666666667W4+0.142857142857143W5+1W6<=1
0.769230769230769W1+0.75W2+0.3333333333333W3+1W4+0.3333333333333W5+0.8W6<=1
0.425531914893617W1+1W2+0.285714285714286W3+0.555555555555W4+0.2W5+0.6W6<=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end
```

Germany – OSD-CCR

```
! Germany OSD-CCR STEP3

max 0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6

st

0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6<=1
0.615384615384615W1+0.60416666666667W2+1W3+0.206185567010309W4+1W5+0.1W6<=1
0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6<=1
1W1+0.833333333333333W2+0.2W3+0.26666666666667W4+0.142857142857143W5+1W6<=1
0.769230769230769W1+0.75W2+0.3333333333333W3+1W4+0.3333333333333W5+0.8W6<=1
0.425531914893617W1+1W2+0.285714285714286W3+0.555555555555W4+0.2W5+0.6W6<=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end
```

UK – OSD-CCR

```
! UK OSD-CCR STEP3

max
1W1+0.833333333333333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6

st

0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6<=1
0.615384615384615W1+0.60416666666667W2+1W3+0.206185567010309W4+1W5+0.1W6<=1
0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6<=1
1W1+0.833333333333333W2+0.2W3+0.26666666666667W4+0.142857142857143W5+1W6<=1
0.769230769230769W1+0.75W2+0.3333333333333W3+1W4+0.3333333333333W5+0.8W6<=1
0.425531914893617W1+1W2+0.285714285714286W3+0.555555555555W4+0.2W5+0.6W6<=1

W1>0
W2>0
W3>0
```

```
W4>0
W5>0
W6>0

end
```

Portugal – OSD-CCR

```
! Portugal OSD-CCR STEP3

max
0.769230769230769W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W
6

st

0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6<=1
0.615384615384615W1+0.60416666666667W2+1W3+0.206185567010309W4+1W5+0.1W6<=1
0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6<=1
1W1+0.833333333333333W2+0.2W3+0.26666666666667W4+0.142857142857143W5+1W6<=1
0.769230769230769W1+0.75W2+0.3333333333333W3+1W4+0.3333333333333W5+0.8W6<=1
0.425531914893617W1+1W2+0.285714285714286W3+0.555555555555W4+0.2W5+0.6W6<=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end
```

France – OSD-CCR

```
! France OSD-CCR STEP3

max
0.425531914893617W1+1W2+0.285714285714286W3+0.55555555555555W4+0.2W5+0.6W6

st

0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6<=1
0.615384615384615W1+0.60416666666667W2+1W3+0.206185567010309W4+1W5+0.1W6<=1
0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6<=1
1W1+0.833333333333333W2+0.2W3+0.26666666666667W4+0.142857142857143W5+1W6<=1
0.769230769230769W1+0.75W2+0.3333333333333W3+1W4+0.3333333333333W5+0.8W6<=1
0.425531914893617W1+1W2+0.285714285714286W3+0.555555555555W4+0.2W5+0.6W6<=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end
```

Results Italy – OSD-CCR

```
 LP OPTIMUM FOUND AT STEP      3

        OBJECTIVE FUNCTION VALUE

        1)     0.9891357
```

```
 VARIABLE        VALUE          REDUCED COST
       W1         0.279341          0.000000
       W2         0.779003          0.000000
       W3         0.357450          0.000000
       W4         0.000000          0.117999
       W5         0.000000          0.133906
       W6         0.000000          0.095325


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.010864          0.000000
       3)         0.000000          0.084195
       4)         0.199776          0.000000
       5)         0.000000          0.109852
       6)         0.081720          0.000000
       7)         0.000000          0.795089
       8)         0.279341          0.000000
       9)         0.779003          0.000000
      10)         0.357450          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000
      13)         0.000000          0.000000

NO. ITERATIONS=       3
```

Results Belgium – OSD-CCR

```
LP OPTIMUM FOUND AT STEP      1

       OBJECTIVE FUNCTION VALUE

       1)      1.000000

 VARIABLE        VALUE          REDUCED COST
       W1         0.000000          0.000000
       W2         0.000000          0.000000
       W3         1.000000          0.000000
       W4         0.000000          0.000000
       W5         0.000000          0.000000
       W6         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.666667          0.000000
       3)         0.000000          1.000000
       4)         0.500000          0.000000
       5)         0.800000          0.000000
       6)         0.666667          0.000000
       7)         0.714286          0.000000
       8)         0.000000          0.000000
       9)         0.000000          0.000000
      10)         1.000000          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000
      13)         0.000000          0.000000

NO. ITERATIONS=       1
```

Results Germany – OSD-CCR

```
LP OPTIMUM FOUND AT STEP      2
```

```
       OBJECTIVE FUNCTION VALUE

       1)      1.000000

 VARIABLE        VALUE          REDUCED COST
       W1         0.000000          0.000000
       W2         0.000000          0.000000
       W3         0.833333          0.000000
       W4         0.000000          0.000000
       W5         0.000000          0.000000
       W6         0.833333          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.305556          0.000000
       3)         0.083333          0.000000
       4)         0.000000          1.000000
       5)         0.000000          0.000000
       6)         0.055556          0.000000
       7)         0.261905          0.000000
       8)         0.000000          0.000000
       9)         0.000000          0.000000
      10)         0.833333          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000
      13)         0.833333          0.000000

NO. ITERATIONS=       2
```

Results UK – OSD-CCR

```
LP OPTIMUM FOUND AT STEP      1

       OBJECTIVE FUNCTION VALUE

       1)      1.000000

 VARIABLE        VALUE          REDUCED COST
       W1         1.000000          0.000000
       W2         0.000000          0.000000
       W3         0.000000          0.000000
       W4         0.000000          0.000000
       W5         0.000000          0.000000
       W6         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.500000          0.000000
       3)         0.384615          0.000000
       4)         0.518072          0.000000
       5)         0.000000          1.000000
       6)         0.230769          0.000000
       7)         0.574468          0.000000
       8)         1.000000          0.000000
       9)         0.000000          0.000000
      10)         0.000000          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000
      13)         0.000000          0.000000

NO. ITERATIONS=       1
```

Results Portugal – OSD-CCR

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      1.000000

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          0.000000
        W2         0.000000          0.000000
        W3         0.000000          0.000000
        W4         1.000000          0.000000
        W5         0.000000          0.000000
        W6         0.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.629630          0.000000
        3)         0.793814          0.000000
        4)         0.722222          0.000000
        5)         0.733333          0.000000
        6)         0.000000          1.000000
        7)         0.444444          0.000000
        8)         0.000000          0.000000
        9)         0.000000          0.000000
       10)         0.000000          0.000000
       11)         1.000000          0.000000
       12)         0.000000          0.000000
       13)         0.000000          0.000000

 NO. ITERATIONS=       1
```

Results France – OSD-CCR

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      1.000000

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          0.000000
        W2         1.000000          0.000000
        W3         0.000000          0.000000
        W4         0.000000          0.000000
        W5         0.000000          0.000000
        W6         0.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.062500          0.000000
        3)         0.395833          0.000000
        4)         0.375000          0.000000
        5)         0.166667          0.000000
        6)         0.250000          0.000000
        7)         0.000000          1.000000
        8)         0.000000          0.000000
        9)         1.000000          0.000000
       10)         0.000000          0.000000
       11)         0.000000          0.000000
```

```
      12)         0.000000          0.000000
      13)         0.000000          0.000000

 NO. ITERATIONS=       1
```

Lindo programs and output for OSD-IP Model

Italy – OSD-IP

```
! Italy OSD-IP STEP3

min Z1+Z2+Z3+Z4+Z5+Z6

st

0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6-0.5W1-
0.9375W2-0.333333333333333W3-0.37037037037037W4-0.125W5-0.5W6-Z1<=0
0.615384615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6-
0.5W1-0.9375W2-0.333333333333333W3-0.37037037037037W4-0.125W5-0.5W6-Z2<=0
0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6-0.5W1-
0.9375W2-0.333333333333333W3-0.37037037037037W4-0.125W5-0.5W6-Z3<=0
1W1+0.833333333333333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6-
0.5W1-0.9375W2-0.333333333333333W3-0.37037037037037W4-0.125W5-0.5W6-Z4<=0
0.769230769230769W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W
6-0.5W1-0.9375W2-0.333333333333333W3-0.37037037037037W4-0.125W5-0.5W6-Z5<=0
0.425531914893617W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6
-0.5W1-0.9375W2-0.333333333333333W3-0.37037037037037W4-0.125W5-0.5W6-Z6<=0

W1+W2+W3+W4+W5+W6=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
```

Belgium – OSD-IP

```
! Belgium OSD-IP STEP3

min Z1+Z2+Z3+Z4+Z5+Z6

st

0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6-
0.615384615384615W1-0.604166666666667W2-1W3-0.206185567010309W4-1W5-0.1W6-
Z1<=0
0.615384615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6-
0.615384615384615W1-0.604166666666667W2-1W3-0.206185567010309W4-1W5-0.1W6-
Z2<=0
```

```
0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6-
0.615384615384615W1-0.604166666666667W2-1W3-0.206185567010309W4-1W5-0.1W6-
Z3<=0
1W1+0.833333333333333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6-
0.615384615384615W1-0.604166666666667W2-1W3-0.206185567010309W4-1W5-0.1W6-
Z4<=0
0.769230769230769W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W
6-0.615384615384615W1-0.604166666666667W2-1W3-0.206185567010309W4-1W5-
0.1W6-Z5<=0
0.425531914893617W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6
-0.615384615384615W1-0.604166666666667W2-1W3-0.206185567010309W4-1W5-0.1W6-
Z6<=0

W1+W2+W3+W4+W5+W6=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
```

Germany – OSD-IP

```
! Germany OSD-IP STEP3

min Z1+Z2+Z3+Z4+Z5+Z6

st

0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6-
0.481927710843373W1-0.625W2-0.5W3-0.277777777777778W4-0.25W5-0.7W6-Z1<=0
0.615384615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6-
0.481927710843373W1-0.625W2-0.5W3-0.277777777777778W4-0.25W5-0.7W6-Z2<=0
0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6-
0.481927710843373W1-0.625W2-0.5W3-0.277777777777778W4-0.25W5-0.7W6-Z3<=0
1W1+0.833333333333333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6-
0.481927710843373W1-0.625W2-0.5W3-0.277777777777778W4-0.25W5-0.7W6-Z4<=0
0.769230769230769W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W
6-0.481927710843373W1-0.625W2-0.5W3-0.277777777777778W4-0.25W5-0.7W6-Z5<=0
0.425531914893617W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6
-0.481927710843373W1-0.625W2-0.5W3-0.277777777777778W4-0.25W5-0.7W6-Z6<=0

W1+W2+W3+W4+W5+W6=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end
```

```
Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
```

UK – OSD-IP

```
! UK OSD-IP STEP3

min Z1+Z2+Z3+Z4+Z5+Z6

st

0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6-1W1-
0.833333333333333W2-0.2W3-0.266666666666667W4-0.142857142857143W5-1W6-Z1<=0
0.615384615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6-
1W1-0.833333333333333W2-0.2W3-0.266666666666667W4-0.142857142857143W5-1W6-
Z2<=0
0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6-1W1-
0.833333333333333W2-0.2W3-0.266666666666667W4-0.142857142857143W5-1W6-Z3<=0
1W1+0.833333333333333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6-
1W1-0.833333333333333W2-0.2W3-0.266666666666667W4-0.142857142857143W5-1W6-
Z4<=0
0.769230769230769W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W
6-1W1-0.833333333333333W2-0.2W3-0.266666666666667W4-0.142857142857143W5-
1W6-Z5<=0
0.425531914893617W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6
-1W1-0.833333333333333W2-0.2W3-0.266666666666667W4-0.142857142857143W5-1W6-
Z6<=0

W1+W2+W3+W4+W5+W6=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
```

Portugal – OSD-IP

```
! Portugal OSD-IP STEP3

min Z1+Z2+Z3+Z4+Z5+Z6

st

0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6-
0.769230769230769W1-0.75W2-0.333333333333333W3-1W4-0.333333333333333W5-
0.8W6-Z1<=0
```

```
0.615384615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6-
0.769230769230769W1-0.75W2-0.333333333333333W3-1W4-0.333333333333333W5-
0.8W6-Z2<=0
0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6-
0.769230769230769W1-0.75W2-0.333333333333333W3-1W4-0.333333333333333W5-
0.8W6-Z3<=0
1W1+0.833333333333333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6-
0.769230769230769W1-0.75W2-0.333333333333333W3-1W4-0.333333333333333W5-
0.8W6-Z4<=0
0.769230769230769W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W
6-0.769230769230769W1-0.75W2-0.333333333333333W3-1W4-0.333333333333333W5-
0.8W6-Z5<=0
0.425531914893617W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6
-0.769230769230769W1-0.75W2-0.333333333333333W3-1W4-0.333333333333333W5-
0.8W6-Z6<=0

W1+W2+W3+W4+W5+W6=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
```

France – OSD-IP

```
! France OSD-IP STEP3

min Z1+Z2+Z3+Z4+Z5+Z6

st

0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6-
0.425531914893617W1-1W2-0.285714285714286W3-0.555555555555555W4-0.2W5-
0.6W6-Z1<=0
0.615384615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6-
0.425531914893617W1-1W2-0.285714285714286W3-0.555555555555555W4-0.2W5-
0.6W6-Z2<=0
0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6-
0.425531914893617W1-1W2-0.285714285714286W3-0.555555555555555W4-0.2W5-
0.6W6-Z3<=0
1W1+0.833333333333333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6-
0.425531914893617W1-1W2-0.285714285714286W3-0.555555555555555W4-0.2W5-
0.6W6-Z4<=0
0.769230769230769W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W
6-0.425531914893617W1-1W2-0.285714285714286W3-0.555555555555555W4-0.2W5-
0.6W6-Z5<=0
0.425531914893617W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6
-0.425531914893617W1-1W2-0.285714285714286W3-0.555555555555555W4-0.2W5-
0.6W6-Z6<=0

W1+W2+W3+W4+W5+W6=1
```

```
W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
```

Results Italy – OSD-IP

```
 LP OPTIMUM FOUND AT STEP      5
 OBJECTIVE VALUE =  0.963884965E-02

 FIX ALL VARS.(    5)  WITH RC >  0.858153
 SET       Z6 TO >=     1 AT    1, BND=  -1.000     TWIN=-0.1000E+31     24

 NEW INTEGER SOLUTION OF    1.00000000     AT BRANCH      1 PIVOT      24
 BOUND ON OPTIMUM: 0.8677918
 DELETE       Z6 AT LEVEL     1
 ENUMERATION COMPLETE. BRANCHES=     1 PIVOTS=      24

 LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)      1.000000

  VARIABLE        VALUE          REDUCED COST
        Z1         0.000000          1.000000
        Z2         0.000000          1.000000
        Z3         0.000000          1.000000
        Z4         0.000000          1.000000
        Z5         0.000000          1.000000
        Z6         1.000000          1.000000
        W1         0.126160          0.000000
        W2         0.644828          0.000000
        W3         0.000000          0.000000
        W4         0.000000          0.000000
        W5         0.229012          0.000000
        W6         0.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          0.000000
        3)         0.000000          0.000000
        4)         0.175162          0.000000
        5)         0.000000          0.000000
        6)         0.039228          0.000000
        7)         0.951917          0.000000
        8)         0.000000          0.000000
        9)         0.126160          0.000000
```

```
      10)         0.644828          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000
      13)         0.229012          0.000000
      14)         0.000000          0.000000

 NO. ITERATIONS=      25
 BRANCHES=    1 DETERM.=  1.000E    0
```

Results Belgium – OSD-IP

```
 LP OPTIMUM FOUND AT STEP      2
 OBJECTIVE VALUE =  0.000000000E+00

 FIX ALL VARS.(    6)  WITH RC >   1.00000

 NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       2
 BOUND ON OPTIMUM: 0.0000000E+00
 ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       2

 LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)     0.0000000E+00

  VARIABLE        VALUE          REDUCED COST
        Z1         0.000000          1.000000
        Z2         0.000000          1.000000
        Z3         0.000000          1.000000
        Z4         0.000000          1.000000
        Z5         0.000000          1.000000
        Z6         0.000000          1.000000
        W1         0.690265          0.000000
        W2         0.000000          0.000000
        W3         0.000000          0.000000
        W4         0.000000          0.000000
        W5         0.309734          0.000000
        W6         0.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.350664          0.000000
        3)         0.000000          0.000000
        4)         0.324422          0.000000
        5)         0.000000          0.000000
        6)         0.100295          0.000000
        7)         0.378836          0.000000
        8)         0.000000          0.000000
        9)         0.690265          0.000000
       10)         0.000000          0.000000
       11)         0.000000          0.000000
       12)         0.000000          0.000000
       13)         0.309734          0.000000
       14)         0.000000          0.000000

 NO. ITERATIONS=       2
 BRANCHES=    0 DETERM.=  1.000E    0
```

Results Germany – OSD-IP

```
 LP OPTIMUM FOUND AT STEP      3
 OBJECTIVE VALUE =  0.000000000E+00
```

```
 FIX ALL VARS.(    6)  WITH RC >   1.00000

 NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       3
 BOUND ON OPTIMUM: 0.0000000E+00
 ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       3

 LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)      0.0000000E+00

  VARIABLE        VALUE          REDUCED COST
        Z1         0.000000          1.000000
        Z2         0.000000          1.000000
        Z3         0.000000          1.000000
        Z4         0.000000          1.000000
        Z5         0.000000          1.000000
        Z6         0.000000          1.000000
        W1         0.000000          0.000000
        W2         0.000000          0.000000
        W3         0.500000          0.000000
        W4         0.000000          0.000000
        W5         0.000000          0.000000
        W6         0.500000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.183333          0.000000
        3)         0.050000          0.000000
        4)         0.000000          0.000000
        5)         0.000000          0.000000
        6)         0.033333          0.000000
        7)         0.157143          0.000000
        8)         0.000000          0.000000
        9)         0.000000          0.000000
       10)         0.000000          0.000000
       11)         0.500000          0.000000
       12)         0.000000          0.000000
       13)         0.000000          0.000000
       14)         0.500000          0.000000

 NO. ITERATIONS=       3
 BRANCHES=    0 DETERM.=  1.000E    0
```

Results UK – OSD-IP

```
 LP OPTIMUM FOUND AT STEP      1
 OBJECTIVE VALUE =  0.000000000E+00

 FIX ALL VARS.(    6)  WITH RC >   1.00000

 NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       1
 BOUND ON OPTIMUM: 0.0000000E+00
 ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       1

 LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE
```

```
       1)     0.0000000E+00

 VARIABLE        VALUE          REDUCED COST
       Z1         0.000000          1.000000
       Z2         0.000000          1.000000
       Z3         0.000000          1.000000
       Z4         0.000000          1.000000
       Z5         0.000000          1.000000
       Z6         0.000000          1.000000
       W1         1.000000          0.000000
       W2         0.000000          0.000000
       W3         0.000000          0.000000
       W4         0.000000          0.000000
       W5         0.000000          0.000000
       W6         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.500000          0.000000
       3)         0.384615          0.000000
       4)         0.518072          0.000000
       5)         0.000000          0.000000
       6)         0.230769          0.000000
       7)         0.574468          0.000000
       8)         0.000000          0.000000
       9)         1.000000          0.000000
      10)         0.000000          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000
      13)         0.000000          0.000000
      14)         0.000000          0.000000

 NO. ITERATIONS=       1
 BRANCHES=    0 DETERM.=  1.000E    0
```

Results Portugal – OSD-IP

```
 LP OPTIMUM FOUND AT STEP      2
 OBJECTIVE VALUE =  0.000000000E+00

 FIX ALL VARS.(    6)  WITH RC >   1.00000

 NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       2
 BOUND ON OPTIMUM: 0.0000000E+00
 ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       2

 LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

       1)     0.0000000E+00

 VARIABLE        VALUE          REDUCED COST
       Z1         0.000000          1.000000
       Z2         0.000000          1.000000
       Z3         0.000000          1.000000
       Z4         0.000000          1.000000
       Z5         0.000000          1.000000
       Z6         0.000000          1.000000
       W1         0.760638          0.000000
       W2         0.000000          0.000000
       W3         0.000000          0.000000
```

```
       W4         0.239362          0.000000
       W5         0.000000          0.000000
       W6         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.355496          0.000000
       3)         0.307030          0.000000
       4)         0.391406          0.000000
       5)         0.000000          0.000000
       6)         0.000000          0.000000
       7)         0.367813          0.000000
       8)         0.000000          0.000000
       9)         0.760638          0.000000
      10)         0.000000          0.000000
      11)         0.000000          0.000000
      12)         0.239362          0.000000
      13)         0.000000          0.000000
      14)         0.000000          0.000000

 NO. ITERATIONS=       2
 BRANCHES=    0 DETERM.=  1.000E    0
```

Results France – OSD-IP

```
 LP OPTIMUM FOUND AT STEP      3
 OBJECTIVE VALUE =  0.000000000E+00

 FIX ALL VARS.(    6)  WITH RC >   1.00000

 NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       3
 BOUND ON OPTIMUM: 0.0000000E+00
 ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       3

 LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)      0.0000000E+00

  VARIABLE        VALUE          REDUCED COST
        Z1         0.000000          1.000000
        Z2         0.000000          1.000000
        Z3         0.000000          1.000000
        Z4         0.000000          1.000000
        Z5         0.000000          1.000000
        Z6         0.000000          1.000000
        W1         0.249125          0.000000
        W2         0.603858          0.000000
        W3         0.000000          0.000000
        W4         0.147016          0.000000
        W5         0.000000          0.000000
        W6         0.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.046415          0.000000
        3)         0.243093          0.000000
        4)         0.253235          0.000000
        5)         0.000000          0.000000
        6)         0.000000          0.000000
        7)         0.000000          0.000000
```

```
       8)         0.000000          0.000000
       9)         0.249125          0.000000
      10)         0.603858          0.000000
      11)         0.000000          0.000000
      12)         0.147016          0.000000
      13)         0.000000          0.000000
      14)         0.000000          0.000000

 NO. ITERATIONS=       3
 BRANCHES=    0 DETERM.=  1.000E    0
```

Lindo programs and output for OSD-DA model

Italy – OSD-DA

```
! Italy OSD-DA STEP3

min
0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6+0.61538
4615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6+0.481927
710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6+1W1+0.8333333333
33333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6+0.769230769230769
W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W6+0.4255319148936
17W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6

st

0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end
```

Belgium – OSD-DA

```
! Belgium OSD-DA STEP3

min
0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6+0.61538
4615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6+0.481927
710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6+1W1+0.8333333333
33333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6+0.769230769230769
W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W6+0.4255319148936
17W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6

st

0.615384615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end
```

Germany – OSD-DA

```
! Germany OSD-DA STEP3

min
0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6+0.61538
4615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6+0.481927
710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6+1W1+0.8333333333
33333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6+0.769230769230769
W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W6+0.4255319148936
17W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6

st

0.481927710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end
```

UK – OSD-DA

```
! UK OSD-DA STEP3

min
0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6+0.61538
4615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6+0.481927
710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6+1W1+0.8333333333
33333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6+0.769230769230769
W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W6+0.4255319148936
17W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6

st

1W1+0.833333333333333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end
```

Portugal – OSD-DA

```
! Portugal OSD-DA STEP3

min
0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6+0.61538
4615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6+0.481927
710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6+1W1+0.8333333333
33333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6+0.769230769230769
W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W6+0.4255319148936
17W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6

st
```

```
0.769230769230769W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W
6=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end
```

France – OSD-DA

```
! France OSD-DA STEP3

min
0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6+0.61538
4615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6+0.481927
710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6+1W1+0.8333333333
33333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6+0.769230769230769
W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W6+0.4255319148936
17W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6

st

0.425531914893617W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6
=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end
```

Results Italy – OSD-DA

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      5.066667

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          1.258742
        W2         1.066667          0.000000
        W3         0.000000          0.963492
        W4         0.000000          0.800013
        W5         0.000000          1.417857
        W6         0.000000          1.166667


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -5.066667
        3)         0.000000          0.000000
        4)         1.066667          0.000000
        5)         0.000000          0.000000
        6)         0.000000          0.000000
        7)         0.000000          0.000000
        8)         0.000000          0.000000
```

```
 NO. ITERATIONS=       1
```

Results Belgium – OSD-DA

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      2.051191

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          2.529804
        W2         0.000000          3.510739
        W3         0.000000          0.601190
        W4         0.000000          2.253630
        W5         1.000000          0.000000
        W6         0.000000          3.494881


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -2.051191
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)         0.000000          0.000000
        6)         0.000000          0.000000
        7)         1.000000          0.000000
        8)         0.000000          0.000000

 NO. ITERATIONS=       1
```

Results Germany – OSD-DA

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      5.285714

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          1.244743
        W2         0.000000          1.446429
        W3         0.000000          0.009524
        W4         0.000000          1.208302
        W5         0.000000          0.729762
        W6         1.428571          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -5.285714
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)         0.000000          0.000000
        6)         0.000000          0.000000
        7)         0.000000          0.000000
        8)         1.428571          0.000000

 NO. ITERATIONS=       1
```

Results UK – OSD-DA

```
 LP OPTIMUM FOUND AT STEP      1
```

```
        OBJECTIVE FUNCTION VALUE

        1)      3.700000

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          0.092075
        W2         0.000000          1.666667
        W3         0.000000          1.912381
        W4         0.000000          1.689889
        W5         0.000000          1.522619
        W6         1.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -3.700000
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)         0.000000          0.000000
        6)         0.000000          0.000000
        7)         0.000000          0.000000
        8)         1.000000          0.000000

 NO. ITERATIONS=       1
```

Results Portugal – OSD-DA

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      2.676556

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          1.733186
        W2         0.000000          2.742583
        W3         0.000000          1.760196
        W4         1.000000          0.000000
        W5         0.000000          1.159005
        W6         0.000000          1.558755


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -2.676556
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)         0.000000          0.000000
        6)         1.000000          0.000000
        7)         0.000000          0.000000
        8)         0.000000          0.000000

 NO. ITERATIONS=       1
```

Results France – OSD-DA

```
! France OSD-DA STEP3

min
0.5W1+0.9375W2+0.333333333333333W3+0.37037037037037W4+0.125W5+0.5W6+0.61538
4615384615W1+0.604166666666667W2+1W3+0.206185567010309W4+1W5+0.1W6+0.481927
710843373W1+0.625W2+0.5W3+0.277777777777778W4+0.25W5+0.7W6+1W1+0.8333333333
33333W2+0.2W3+0.266666666666667W4+0.142857142857143W5+1W6+0.769230769230769
```

```
W1+0.75W2+0.333333333333333W3+1W4+0.333333333333333W5+0.8W6+0.4255319148936
17W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6

st

0.425531914893617W1+1W2+0.285714285714286W3+0.555555555555555W4+0.2W5+0.6W6
=1

W1>0
W2>0
W3>0
W4>0
W5>0
W6>0

end
```

Power Plant Site Selection [STEW92]

Seven Steps using OSD-CCR

Step 1a - Collect Measures

Location	(I) Manpower	(O) PowGen	(I) ConstructCost	(I) MaintenCost	(I) VillagesEvac	(O) SafetyLevel
Italy	80	90	600	54	8	5
Belgium	65	58	200	97	1	1
Germany	83	60	400	72	4	7
UK	40	80	1000	75	7	10
Portugal	52	72	600	20	3	8
France	94	96	700	36	5	6

Step 1b - Orient Measures

Alternative	C1	C2	C3	C4	C5	C6
Italy	0.013	90.000	0.002	0.019	0.125	5.000
Belgium	0.015	58.000	0.005	0.010	1.000	1.000
Germany	0.012	60.000	0.003	0.014	0.250	7.000
UK	0.025	80.000	0.001	0.013	0.143	10.000
Portugal	0.019	72.000	0.002	0.050	0.333	8.000
France	0.011	96.000	0.001	0.028	0.200	6.000

Step 2 - Level Scales

Alternative	C1	C2	C3	C4	C5	C6	Total
Italy	0.500	0.938	0.333	0.370	0.125	0.500	2.766
Belgium	0.615	0.604	1.000	0.206	1.000	0.100	3.526
Germany	0.482	0.625	0.500	0.278	0.250	0.700	2.835
UK	1.000	0.833	0.200	0.267	0.143	1.000	3.443
Portugal	0.769	0.750	0.333	1.000	0.333	0.800	3.986
France	0.426	1.000	0.286	0.556	0.200	0.600	3.067

Step 3 - Choose Weights

OSD-CCR RAW RESULTS

Alternative	W1	W2	W3	W4	W5	W6	OSD-CCR SC(
Italy	0.279	0.779	0.357				0.989
Belgium	0.279	0.779	0.357				1.000
Germany		0.214	0.794	0.057		0.648	1.000
UK		0.214	0.794	0.057		0.648	1.000
Portugal		0.214	0.794	0.057		0.648	1.000
France		0.548	0.616	0.060		0.404	1.000

OSD-CCR NORM. RESULTS

Alternative	W1	W2	W3	W4	W5	W6
Italy	0.197	0.550	0.252			
Belgium	0.197	0.550	0.252			
Germany		0.125	0.464	0.033		0.378
UK		0.125	0.464	0.033		0.378
Portugal		0.125	0.464	0.033		0.378
France		0.337	0.378	0.037		0.248

Step 4 - Stack Weights

OSD-CCR STACKED WEIGHTS

W1	W2	W3	W4	W5	W6
0.395	1.812	2.275	0.136	0.000	1.383

Step 5 - Calculate Weight Importance Factor

OSD-CCR WIF

W1	W2	W3	W4	W5	W6
7%	30%	38%	2%	0%	23%

Step 6 - Calculate Scores

OSD-CCR C-SCORES

Alternative	C-Score
Italy	0.566
Belgium	0.630
Germany	0.578
UK	0.630
Portugal	0.611
France	0.589

Step 7 - Rank Alternatives

OSD-CCR C-RANKS

Alternative	C-Rank
Italy	6
Belgium	2
Germany	5
UK	1
Portugal	3
France	4

Power Plant Site Selection [STEW92]

Seven Steps using OSD-IP

Step 1a - Collect Measures

Location	(I) Manpower	(O) PowGen	(I) ConstructCost	(I) MaintenCost	(I) VillagesEvac	(O) SafetyLevel
Italy	80	90	600	54	8	5
Belgium	65	58	200	97	1	1
Germany	83	60	400	72	4	7
UK	40	80	1000	75	7	10
Portugal	52	72	600	20	3	8
France	94	96	700	36	5	6

Step 1b - Orient Measures

Alternative	C1	C2	C3	C4	C5	C6
Italy	0.013	90.000	0.002	0.019	0.125	5.000
Belgium	0.015	58.000	0.005	0.010	1.000	1.000
Germany	0.012	60.000	0.003	0.014	0.250	7.000
UK	0.025	80.000	0.001	0.013	0.143	10.000
Portugal	0.019	72.000	0.002	0.050	0.333	8.000
France	0.011	96.000	0.001	0.028	0.200	6.000

Step 2 - Level Scales

Alternative	C1	C2	C3	C4	C5	C6	Total
Italy	0.500	0.938	0.333	0.370	0.125	0.500	2.766
Belgium	0.615	0.604	1.000	0.206	1.000	0.100	3.526
Germany	0.482	0.625	0.500	0.278	0.250	0.700	2.835
UK	1.000	0.833	0.200	0.267	0.143	1.000	3.443
Portugal	0.769	0.750	0.333	1.000	0.333	0.800	3.986
France	0.426	1.000	0.286	0.556	0.200	0.600	3.067

Step 3 - Choose Weights

OSD-IP RAW RESULTS

Alternative	W1	W2	W3	W4	W5	W6	No.Above	Max Rank
Italy	0.126	0.645			0.229		1	2
Belgium	0.690				0.310		0	1
Germany			0.500			0.500	0	1
UK	1.000						0	1
Portugal	0.761			0.239			0	1
France	0.249	0.604		0.147			0	1

OSD-IP NORM. RESULTS

Alternative	W1	W2	W3	W4	W5	W6
Italy	0.126	0.645	0.000	0.000	0.229	0.000
Belgium	0.690	0.000	0.000	0.000	0.310	0.000
Germany	0.000	0.000	0.500	0.000	0.000	0.500
UK	1.000	0.000	0.000	0.000	0.000	0.000
Portugal	0.761	0.000	0.000	0.239	0.000	0.000
France	0.249	0.604	0.000	0.147	0.000	0.000

Step 4 - Stack Weights

OSD-IP STACKED WEIGHTS

	W1	W2	W3	W4	W5	W6
	2.826	1.249	0.500	0.386	0.539	0.500

Step 5 - Calculate Weight Importance Factor

OSD-IP WIF

	W1	W2	W3	W4	W5	W6
	47%	21%	8%	6%	9%	8%

Step 6 - Calculate Scores

OSD-IP C-SCORES

Alternative	C-Score
Italy	0.535
Belgium	0.610
Germany	0.497
UK	0.774
Portugal	0.707
France	0.536

Step 7 - Rank Alternatives

OSD-IP C-RANKS

Alternative	C-Rank
Italy	5
Belgium	3
Germany	6
UK	1
Portugal	2
France	4

Power Plant Site Selection [STEW92]

Seven Steps using OSD-DA

Step 1a - Collect Measures

Location	(I) Manpower	(O) PowGen	(I) ConstructCost	(I) MaintenCost	(I) VillagesEvac	(O) SafetyLevel
Italy	80	90	600	54	8	5
Belgium	65	58	200	97	1	1
Germany	83	60	400	72	4	7
UK	40	80	1000	75	7	10
Portugal	52	72	600	20	3	8
France	94	96	700	36	5	6

Step 1b - Orient Measures

Alternative	C1	C2	C3	C4	C5	C6
Italy	0.013	90.000	0.002	0.019	0.125	5.000
Belgium	0.015	58.000	0.005	0.010	1.000	1.000
Germany	0.012	60.000	0.003	0.014	0.250	7.000
UK	0.025	80.000	0.001	0.013	0.143	10.000
Portugal	0.019	72.000	0.002	0.050	0.333	8.000
France	0.011	96.000	0.001	0.028	0.200	6.000

Step 2 - Level Scales

Alternative	C1	C2	C3	C4	C5	C6	Total
Italy	0.500	0.938	0.333	0.370	0.125	0.500	2.766
Belgium	0.615	0.604	1.000	0.206	1.000	0.100	3.526
Germany	0.482	0.625	0.500	0.278	0.250	0.700	2.835
UK	1.000	0.833	0.200	0.267	0.143	1.000	3.443
Portugal	0.769	0.750	0.333	1.000	0.333	0.800	3.986
France	0.426	1.000	0.286	0.556	0.200	0.600	3.067

Step 3 - Choose Weights

OSD-DA RAW RESULTS

Alternative	W1	W2	W3	W4	W5	W6	SCORE	Average
Italy		1.067					5.067	0.844
Belgium					1.000		2.051	0.342
Germany						1.429	5.286	0.881
UK						1.000	3.700	0.617
Portugal				1.000			2.677	0.446
France		1.000					4.750	0.792

OSD-DA NORM. RESULTS

Alternative	W1	W2	W3	W4	W5	W6
Italy		1.000				
Belgium					1.000	
Germany						1.000
UK						1.000
Portugal				1.000		
France		1.000				

Step 4 - Stack Weights

OSD-DA STACKED WEIGHTS

W1	W2	W3	W4	W5	W6
0	2	0	1	1	2

Step 5 - Calculate Weight Importance Factor

OSD-DA WIF

W1	W2	W3	W4	W5	W6
0%	33%	0%	17%	17%	33%

Step 6 - Calculate Scores

OSD-DA C-SCORES

Alternative	C-Score
Italy	0.562
Belgium	0.436
Germany	0.530
UK	0.679
Portugal	0.739
France	0.659

Step 7 - Rank Alternatives

OSD-DA C-RANKS

Alternative	C-Rank
Italy	4
Belgium	6
Germany	5
UK	2
Portugal	1
France	3

APPENDIX B: GGA RANKING OF RGPS: ALL WEIGHTS, UNIQUE WEIGHTS & SCORES

Evaluation of 200 RGPs on 9 Criteria for GGA

ProDEA Normalized Weights Results

DMU	DEA Score	U(1)	U(2)	U(3)	U(4)	U(5)	U(6)	U(7)	U(8)	U(9)	Sum
1	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
2	1.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
3	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
4	0.924	0.374	0.000	0.234	0.209	0.000	0.000	0.000	0.183	0.000	1
5	0.924	0.374	0.000	0.234	0.209	0.000	0.000	0.000	0.183	0.000	1
6	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
7	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1
8	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
9	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
10	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
11	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
12	0.903	0.242	0.000	0.000	0.270	0.176	0.000	0.000	0.000	0.312	1
13	0.915	0.000	0.000	0.000	0.095	0.386	0.000	0.000	0.519	0.000	1
14	0.918	0.306	0.000	0.000	0.000	0.000	0.000	0.000	0.300	0.394	1
15	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
16	1.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
17	0.915	0.000	0.000	0.000	0.095	0.386	0.000	0.000	0.519	0.000	1
18	0.884	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1
19	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
20	0.924	0.374	0.000	0.234	0.209	0.000	0.000	0.000	0.183	0.000	1
21	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
22	0.924	0.374	0.000	0.234	0.209	0.000	0.000	0.000	0.183	0.000	1
23	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
24	1.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
25	1.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
26	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
27	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
28	0.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
29	0.884	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1
30	0.903	0.242	0.000	0.000	0.270	0.176	0.000	0.000	0.000	0.312	1
31	0.915	0.000	0.000	0.000	0.095	0.386	0.000	0.000	0.519	0.000	1

32	0.900	0.000	0.000	0.000	0.000	0.269	0.000	0.369	0.362	0.000	1
33	0.903	0.242	0.000	0.000	0.270	0.176	0.000	0.000	0.000	0.312	1
34	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
35	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
36	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
37	0.924	0.374	0.000	0.234	0.209	0.000	0.000	0.000	0.183	0.000	1
38	0.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
39	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
40	1.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
41	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
42	1.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
43	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
44	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
45	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
46	0.915	0.000	0.000	0.000	0.095	0.386	0.000	0.000	0.519	0.000	1
47	0.910	0.000	0.000	0.000	0.000	0.243	0.000	0.000	0.327	0.430	1
48	0.903	0.242	0.000	0.000	0.270	0.176	0.000	0.000	0.000	0.312	1
49	1.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
50	1.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
51	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
52	0.915	0.000	0.000	0.120	0.000	0.375	0.000	0.000	0.505	0.000	1
53	0.913	0.323	0.000	0.000	0.360	0.000	0.000	0.000	0.317	0.000	1
54	0.924	0.374	0.000	0.234	0.209	0.000	0.000	0.000	0.183	0.000	1
55	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
56	0.900	0.000	0.000	0.000	0.000	0.269	0.000	0.369	0.362	0.000	1
57	0.918	0.306	0.000	0.000	0.000	0.000	0.000	0.000	0.300	0.394	1
58	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
59	0.825	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
60	0.915	0.000	0.000	0.120	0.000	0.375	0.000	0.000	0.505	0.000	1
61	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
62	1.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
63	0.924	0.374	0.000	0.234	0.209	0.000	0.000	0.000	0.183	0.000	1
64	0.903	0.242	0.000	0.000	0.270	0.176	0.000	0.000	0.000	0.312	1
65	0.919	0.294	0.000	0.000	0.164	0.107	0.000	0.147	0.288	0.000	1
66	0.924	0.374	0.000	0.234	0.209	0.000	0.000	0.000	0.183	0.000	1
67	0.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
68	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
69	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
70	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
71	0.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
72	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
73	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
74	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
75	0.910	0.334	0.000	0.419	0.000	0.000	0.247	0.000	0.000	0.000	1
76	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
77	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
78	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
79	0.889	0.405	0.000	0.000	0.000	0.296	0.299	0.000	0.000	0.000	1
80	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1

81	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
82	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1
83	0.915	0.000	0.000	0.000	0.095	0.386	0.000	0.000	0.519	0.000	1
84	1.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
85	0.884	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1
86	0.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
87	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
88	1.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
89	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1
90	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
91	0.920	0.297	0.000	0.372	0.331	0.000	0.000	0.000	0.000	0.000	1
92	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
93	0.899	0.000	0.092	0.000	0.000	0.387	0.000	0.000	0.521	0.000	1
94	1.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
95	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
96	0.903	0.242	0.000	0.000	0.270	0.176	0.000	0.000	0.000	0.312	1
97	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
98	0.924	0.374	0.000	0.234	0.209	0.000	0.000	0.000	0.183	0.000	1
99	0.920	0.297	0.000	0.372	0.331	0.000	0.000	0.000	0.000	0.000	1
100	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
101	0.889	0.405	0.191	0.000	0.000	0.000	0.000	0.405	0.000	0.000	1
102	0.889	0.405	0.191	0.000	0.000	0.000	0.000	0.405	0.000	0.000	1
103	0.889	0.405	0.000	0.000	0.000	0.296	0.299	0.000	0.000	0.000	1
104	0.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
105	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
106	0.868	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1
107	1.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
108	0.924	0.374	0.000	0.234	0.209	0.000	0.000	0.000	0.183	0.000	1
109	0.924	0.374	0.000	0.234	0.209	0.000	0.000	0.000	0.183	0.000	1
110	0.915	0.000	0.000	0.120	0.000	0.375	0.000	0.000	0.505	0.000	1
111	0.918	0.402	0.000	0.252	0.000	0.000	0.148	0.000	0.197	0.000	1
112	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
113	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
114	0.902	0.362	0.171	0.000	0.000	0.000	0.000	0.000	0.000	0.467	1
115	1.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
116	0.884	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1
117	0.915	0.000	0.000	0.120	0.000	0.375	0.000	0.000	0.505	0.000	1
118	0.868	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1
119	0.902	0.362	0.171	0.000	0.000	0.000	0.000	0.000	0.000	0.467	1
120	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
121	0.920	0.297	0.000	0.372	0.331	0.000	0.000	0.000	0.000	0.000	1
122	0.871	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1
123	0.825	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
124	0.513	0.306	0.000	0.000	0.000	0.000	0.000	0.000	0.300	0.394	1
125	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
126	0.895	0.260	0.000	0.000	0.290	0.190	0.000	0.260	0.000	0.000	1
127	0.513	0.306	0.000	0.000	0.000	0.000	0.000	0.000	0.300	0.394	1
128	0.871	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1
129	0.884	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1

130	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
131	0.508	0.130	0.000	0.000	0.000	0.371	0.000	0.000	0.499	0.000	1
132	0.902	0.362	0.171	0.000	0.000	0.000	0.000	0.000	0.000	0.467	1
133	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
134	0.884	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1
135	0.871	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1
136	0.513	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
137	0.871	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
138	0.868	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1
139	0.871	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
140	0.513	0.306	0.000	0.000	0.000	0.000	0.000	0.000	0.300	0.394	1
141	0.825	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
142	0.884	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1
143	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
144	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
145	0.823	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
146	0.513	0.306	0.000	0.000	0.000	0.000	0.000	0.000	0.300	0.394	1
147	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
148	0.513	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
149	0.493	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
150	0.513	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
151	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
152	0.871	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
153	0.868	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1
154	0.822	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
155	0.868	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1
156	0.884	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1
157	0.899	0.000	0.092	0.000	0.000	0.387	0.000	0.000	0.521	0.000	1
158	0.513	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
159	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
160	0.915	0.000	0.000	0.000	0.095	0.386	0.000	0.000	0.519	0.000	1
161	0.825	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
162	0.825	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
163	0.823	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
164	0.825	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
165	0.868	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1
166	0.822	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
167	0.902	0.362	0.171	0.000	0.000	0.000	0.000	0.000	0.000	0.467	1
168	0.823	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
169	0.871	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1
170	0.492	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
171	0.725	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
172	0.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
173	0.487	0.000	0.000	0.000	0.095	0.386	0.000	0.000	0.519	0.000	1
174	0.825	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
175	0.825	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
176	0.825	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
177	0.513	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
178	0.823	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1

179	0.513	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
180	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
181	0.725	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
182	0.900	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	1
183	0.884	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1
184	0.480	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
185	0.825	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
186	0.871	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1
187	0.871	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
188	0.725	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
189	0.725	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
190	0.513	0.306	0.000	0.000	0.000	0.000	0.000	0.000	0.300	0.394	1
191	1.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1
192	0.868	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1
193	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1
194	0.884	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1
195	0.897	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1
196	0.508	0.130	0.000	0.000	0.000	0.371	0.000	0.000	0.499	0.000	1
197	0.868	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1
198	0.825	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1
199	0.884	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1
200	0.868	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	1
		37.528	27.249	22.844	19.391	24.442	14.993	6.953	23.671	22.928	200
		0.188	**0.136**	**0.114**	**0.097**	**0.122**	**0.075**	**0.035**	**0.118**	**0.115**	
	WIF	**19%**	**14%**	**11%**	**10%**	**12%**	**7%**	**3%**	**12%**	**11%**	

ProDEA Normalized Unique Weights Only

DMU	**DEA Score**	**U(1)**	**U(2)**	**U(3)**	**U(4)**	**U(5)**	**U(6)**	**U(7)**	**U(8)**	**U(9)**	**Sum**
4	0.92	0.37	0.00	0.23	0.21	0.00	0.00	0.00	0.18	0.00	1
5	0.92	0.37	0.00	0.23	0.21	0.00	0.00	0.00	0.18	0.00	1
12	0.90	0.24	0.00	0.00	0.27	0.18	0.00	0.00	0.00	0.31	1
13	0.91	0.00	0.00	0.00	0.10	0.39	0.00	0.00	0.52	0.00	1
14	0.92	0.31	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.39	1
17	0.91	0.00	0.00	0.00	0.10	0.39	0.00	0.00	0.52	0.00	1
18	0.88	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
19	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
20	0.92	0.37	0.00	0.23	0.21	0.00	0.00	0.00	0.18	0.00	1
22	0.92	0.37	0.00	0.23	0.21	0.00	0.00	0.00	0.18	0.00	1
28	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
29	0.88	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
30	0.90	0.24	0.00	0.00	0.27	0.18	0.00	0.00	0.00	0.31	1
31	0.91	0.00	0.00	0.00	0.10	0.39	0.00	0.00	0.52	0.00	1
32	0.90	0.00	0.00	0.00	0.00	0.27	0.00	0.37	0.36	0.00	1
33	0.90	0.24	0.00	0.00	0.27	0.18	0.00	0.00	0.00	0.31	1
34	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1

35	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
36	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
37	0.92	0.37	0.00	0.23	0.21	0.00	0.00	0.00	0.18	0.00	1
38	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
46	0.91	0.00	0.00	0.00	0.10	0.39	0.00	0.00	0.52	0.00	1
47	0.91	0.00	0.00	0.00	0.00	0.24	0.00	0.00	0.33	0.43	1
48	0.90	0.24	0.00	0.00	0.27	0.18	0.00	0.00	0.00	0.31	1
52	0.92	0.00	0.00	0.12	0.00	0.38	0.00	0.00	0.51	0.00	1
53	0.91	0.32	0.00	0.00	0.36	0.00	0.00	0.00	0.32	0.00	1
54	0.92	0.37	0.00	0.23	0.21	0.00	0.00	0.00	0.18	0.00	1
56	0.90	0.00	0.00	0.00	0.00	0.27	0.00	0.37	0.36	0.00	1
57	0.92	0.31	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.39	1
59	0.82	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
60	0.92	0.00	0.00	0.12	0.00	0.38	0.00	0.00	0.51	0.00	1
61	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
63	0.92	0.37	0.00	0.23	0.21	0.00	0.00	0.00	0.18	0.00	1
64	0.90	0.24	0.00	0.00	0.27	0.18	0.00	0.00	0.00	0.31	1
65	0.92	0.29	0.00	0.00	0.16	0.11	0.00	0.15	0.29	0.00	1
66	0.92	0.37	0.00	0.23	0.21	0.00	0.00	0.00	0.18	0.00	1
67	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
68	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
71	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
74	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
75	0.91	0.33	0.00	0.42	0.00	0.00	0.25	0.00	0.00	0.00	1
79	0.89	0.41	0.00	0.00	0.00	0.30	0.30	0.00	0.00	0.00	1
80	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
83	0.91	0.00	0.00	0.00	0.10	0.39	0.00	0.00	0.52	0.00	1
85	0.88	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
86	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
91	0.92	0.30	0.00	0.37	0.33	0.00	0.00	0.00	0.00	0.00	1
93	0.90	0.00	0.09	0.00	0.00	0.39	0.00	0.00	0.52	0.00	1
96	0.90	0.24	0.00	0.00	0.27	0.18	0.00	0.00	0.00	0.31	1
98	0.92	0.37	0.00	0.23	0.21	0.00	0.00	0.00	0.18	0.00	1
99	0.92	0.30	0.00	0.37	0.33	0.00	0.00	0.00	0.00	0.00	1
101	0.89	0.40	0.19	0.00	0.00	0.00	0.00	0.40	0.00	0.00	1
102	0.89	0.40	0.19	0.00	0.00	0.00	0.00	0.40	0.00	0.00	1
103	0.89	0.41	0.00	0.00	0.00	0.30	0.30	0.00	0.00	0.00	1
104	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
106	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1
108	0.92	0.37	0.00	0.23	0.21	0.00	0.00	0.00	0.18	0.00	1
109	0.92	0.37	0.00	0.23	0.21	0.00	0.00	0.00	0.18	0.00	1
110	0.92	0.00	0.00	0.12	0.00	0.38	0.00	0.00	0.51	0.00	1
111	0.92	0.40	0.00	0.25	0.00	0.00	0.15	0.00	0.20	0.00	1
114	0.90	0.36	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.47	1
116	0.88	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
117	0.92	0.00	0.00	0.12	0.00	0.38	0.00	0.00	0.51	0.00	1
118	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1
119	0.90	0.36	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.47	1
121	0.92	0.30	0.00	0.37	0.33	0.00	0.00	0.00	0.00	0.00	1

122	0.87	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1
123	0.82	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
124	0.51	0.31	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.39	1
126	0.90	0.26	0.00	0.00	0.29	0.19	0.00	0.26	0.00	0.00	1
127	0.51	0.31	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.39	1
128	0.87	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1
129	0.88	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
130	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
131	0.51	0.13	0.00	0.00	0.00	0.37	0.00	0.00	0.50	0.00	1
132	0.90	0.36	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.47	1
133	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
134	0.88	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
135	0.87	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1
136	0.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
137	0.87	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
138	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1
139	0.87	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
140	0.51	0.31	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.39	1
141	0.82	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
142	0.88	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
143	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
144	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
145	0.82	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1
146	0.51	0.31	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.39	1
147	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
148	0.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
149	0.49	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
150	0.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
152	0.87	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
153	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1
154	0.82	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1
155	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1
156	0.88	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
157	0.90	0.00	0.09	0.00	0.00	0.39	0.00	0.00	0.52	0.00	1
158	0.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
160	0.91	0.00	0.00	0.00	0.10	0.39	0.00	0.00	0.52	0.00	1
161	0.82	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
162	0.82	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
163	0.82	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1
164	0.82	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
165	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1
166	0.82	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1
167	0.90	0.36	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.47	1
168	0.82	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1
169	0.87	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1
170	0.49	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
171	0.73	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
172	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
173	0.49	0.00	0.00	0.00	0.10	0.39	0.00	0.00	0.52	0.00	1

174	0.82	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
175	0.82	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
176	0.82	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
177	0.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
178	0.82	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1
179	0.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
180	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
181	0.73	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
182	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1
183	0.88	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
184	0.48	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1
185	0.82	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
186	0.87	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1
187	0.87	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
188	0.73	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
189	0.73	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1
190	0.51	0.31	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.39	1
192	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1
194	0.88	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
195	0.90	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1
196	0.51	0.13	0.00	0.00	0.00	0.37	0.00	0.00	0.50	0.00	1
197	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1
198	0.82	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1
199	0.88	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1
200	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1
	Totals	19.53	5.25	19.84	17.39	15.44	11.99	6.95	22.67	20.93	140
		0.14	**0.04**	**0.14**	**0.12**	**0.11**	**0.09**	**0.05**	**0.16**	**0.15**	WIF Unique Weights
		0.19	**0.14**	**0.11**	**0.10**	**0.12**	**0.07**	**0.03**	**0.12**	**0.11**	WIF All Weights

Comparison of Scores

DMU	DEA	WIDE	Expert
6	1.000	0.962	0.966
10	1.000	0.961	0.971
9	1.000	0.946	0.960
92	1.000	0.939	0.922
1	1.000	0.934	0.924
3	1.000	0.928	0.929
78	1.000	0.915	0.911
72	1.000	0.911	0.908
44	1.000	0.908	0.908
45	1.000	0.903	0.885
7	1.000	0.901	0.890
24	1.000	0.895	0.882
26	1.000	0.894	0.896
100	1.000	0.894	0.896

DMU	DEA	WIDE	Expert
8	1.000	0.891	0.893
39	1.000	0.890	0.893
42	1.000	0.887	0.885
82	1.000	0.887	0.866
25	1.000	0.880	0.871
73	1.000	0.876	0.882
97	1.000	0.875	0.814
11	1.000	0.868	0.811
43	1.000	0.866	0.896
20	0.924	0.865	0.851
37	0.924	0.865	0.851
54	0.924	0.865	0.851
63	0.924	0.865	0.851
66	0.924	0.865	0.851

DMU	DEA	WIDE	Expert
98	0.924	0.865	0.851
81	1.000	0.857	0.868
23	1.000	0.838	0.844
4	0.924	0.836	0.785
84	1.000	0.832	0.829
94	1.000	0.832	0.829
113	1.000	0.826	0.843
49	1.000	0.822	0.820
76	1.000	0.818	0.821
109	0.924	0.810	0.761
5	0.924	0.807	0.808
108	0.924	0.807	0.808
50	1.000	0.805	0.780
2	1.000	0.798	0.809
16	1.000	0.789	0.783
15	1.000	0.785	0.802
105	1.000	0.785	0.802
21	1.000	0.785	0.781
51	1.000	0.779	0.810
89	1.000	0.777	0.782
41	1.000	0.776	0.785
22	0.924	0.770	0.770
120	1.000	0.769	0.750
58	1.000	0.768	0.756
14	0.918	0.763	0.763
88	1.000	0.763	0.778
40	1.000	0.762	0.783
52	0.915	0.755	0.766
60	0.915	0.755	0.766
117	0.915	0.755	0.766
31	0.915	0.748	0.759
46	0.915	0.748	0.759
65	0.919	0.743	0.758
12	0.903	0.736	0.756
48	0.903	0.736	0.756
96	0.903	0.736	0.756
70	1.000	0.728	0.766
33	0.903	0.718	0.704
35	0.897	0.717	0.728
61	0.897	0.717	0.728
57	0.918	0.713	0.678
30	0.903	0.706	0.690
55	1.000	0.704	0.732
95	1.000	0.698	0.726
114	0.902	0.698	0.718
71	0.900	0.698	0.725
107	1.000	0.697	0.674
38	0.900	0.693	0.669

DMU	DEA	WIDE	Expert
83	0.915	0.690	0.716
19	0.897	0.690	0.722
34	0.897	0.690	0.722
80	0.897	0.690	0.722
62	1.000	0.687	0.716
91	0.920	0.684	0.720
67	0.900	0.681	0.655
110	0.915	0.680	0.699
87	1.000	0.675	0.705
17	0.915	0.673	0.664
27	1.000	0.671	0.720
64	0.903	0.667	0.667
13	0.915	0.661	0.650
36	0.897	0.660	0.656
99	0.920	0.654	0.661
18	0.884	0.653	0.678
90	1.000	0.648	0.673
115	1.000	0.647	0.623
86	0.900	0.645	0.676
77	1.000	0.640	0.689
119	0.902	0.637	0.637
85	0.884	0.635	0.626
69	1.000	0.633	0.648
32	0.900	0.615	0.649
56	0.900	0.615	0.649
53	0.913	0.607	0.565
111	0.918	0.607	0.594
102	0.889	0.603	0.646
28	0.900	0.603	0.623
75	0.910	0.600	0.592
74	0.897	0.592	0.622
29	0.884	0.588	0.633
79	0.889	0.586	0.594
47	0.910	0.576	0.509
116	0.884	0.570	0.581
101	0.889	0.565	0.608
104	0.900	0.564	0.506
103	0.889	0.561	0.552
106	0.868	0.548	0.545
118	0.868	0.548	0.545
93	0.899	0.536	0.543
59	0.825	0.521	0.538
68	0.897	0.518	0.573
112	1.000	0.515	0.520
125	1.000	0.565	0.563
142	0.884	0.558	0.554
121	0.920	0.554	0.522
132	0.902	0.543	0.520

DMU	DEA	WIDE	Expert
129	0.884	0.541	0.515
126	0.895	0.535	0.533
172	0.900	0.533	0.537
138	0.868	0.531	0.493
160	0.915	0.530	0.592
133	0.897	0.530	0.498
137	0.871	0.523	0.515
157	0.899	0.523	0.490
164	0.825	0.521	0.538
151	1.000	0.521	0.552
156	0.884	0.517	0.588
182	0.900	0.511	0.464
139	0.871	0.506	0.463
152	0.871	0.506	0.463
197	0.868	0.498	0.480
130	0.897	0.497	0.485
186	0.871	0.494	0.512
167	0.902	0.493	0.467
135	0.871	0.492	0.536
180	0.897	0.485	0.484
191	1.000	0.485	0.476
141	0.825	0.483	0.500
161	0.825	0.483	0.500
174	0.825	0.483	0.500
176	0.825	0.483	0.500
153	0.868	0.481	0.453
143	0.897	0.480	0.433
178	0.823	0.471	0.473
147	0.897	0.468	0.508
200	0.868	0.465	0.446
185	0.825	0.464	0.499
134	0.884	0.463	0.477
169	0.871	0.463	0.527
144	0.897	0.461	0.409
166	0.822	0.453	0.421
183	0.884	0.449	0.459
155	0.868	0.448	0.440
159	1.000	0.447	0.491
128	0.871	0.445	0.449
194	0.884	0.434	0.411
123	0.825	0.431	0.459
124	0.513	0.415	0.383
127	0.513	0.415	0.383
140	0.513	0.415	0.383
146	0.513	0.415	0.383
190	0.513	0.415	0.383
162	0.825	0.409	0.409
199	0.884	0.397	0.430

DMU	DEA	WIDE	Expert
154	0.822	0.396	0.382
193	1.000	0.395	0.411
198	0.825	0.388	0.399
165	0.868	0.375	0.393
175	0.825	0.375	0.417
168	0.823	0.363	0.390
192	0.868	0.357	0.363
195	0.897	0.356	0.328
163	0.823	0.350	0.362
145	0.823	0.347	0.343
171	0.725	0.341	0.358
148	0.513	0.340	0.320
150	0.513	0.340	0.320
179	0.513	0.340	0.320
122	0.871	0.334	0.388
158	0.513	0.325	0.311
189	0.725	0.325	0.352
187	0.871	0.307	0.288
196	0.508	0.304	0.299
136	0.513	0.294	0.275
170	0.492	0.293	0.254
131	0.508	0.286	0.274
173	0.487	0.255	0.259
149	0.493	0.253	0.241
188	0.725	0.253	0.283
177	0.513	0.212	0.207
181	0.725	0.195	0.221
184	0.480	0.180	0.196

APPENDIX C: CAPITAL CITY SITE SELECTION LINDO PROGRAMS, OUTPUT & SUMMARY SHEETS

Evaluation of Candidate Sites for New Capital City [COOP00] pp 169-173.

Raw Data:

Alternative	C1	C2	C3	C4
A	5	10	3	5
B	7	10	3	10
C	8	7	10	5
D	4	8	3	10
E	9	4	4	2
F	10	5	10	3
G	4	7	7	8

Lindo programs and output for OSD-CCR model

DMU A – OSD-CCR

```
MAX 0.5 W1 + 1.0 W2 + 0.3 W3 + 0.5 W4

ST

0.5 W1 + 1.0 W2 + 0.3 W3 + 0.5 W4 <= 1
0.7 W1 + 1.0 W2 + 0.3 W3 + 1.0 W4 <= 1
0.8 W1 + 0.7 W2 + 1.0 W3 + 0.5 W4 <= 1
0.4 W1 + 0.8 W2 + 0.3 W3 + 1.0 W4 <= 1
0.9 W1 + 0.4 W2 + 0.4 W3 + 0.2 W4 <= 1
1.0 W1 + 0.5 W2 + 1.0 W3 + 0.3 W4 <= 1
0.4 W1 + 0.7 W2 + 0.7 W3 + 0.8 W4 <= 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU B – OSD-CCR

```
MAX .7 W1 + 1. W2 + .3 W3 + 1. W4

ST
```

```
0.5W1 +1W2   +0.3W3 +0.5W4  <= 1
0.7W1 +1W2   +0.3W3 +1.0W4  <= 1
0.8W1 +.7W2  +1.0W3 +0.5W4  <= 1
0.4W1 +.8W2  +0.3W3 +1.0W4  <= 1
0.9W1 +.4W2  +0.4W3 +0.2W4  <= 1
1.0W1 +.5W2  +1.0W3 +0.3W4  <= 1
0.4W1 +.7W2  +0.7W3 +0.8W4  <= 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU C – OSD-CCR

```
MAX .8 W1 + .7 W2 + 1. W3 + .5 W4

ST

0.5W1 +1W2   +0.3W3 +0.5W4  <= 1
0.7W1 +1W2   +0.3W3 +1.0W4  <= 1
0.8W1 +.7W2  +1.0W3 +0.5W4  <= 1
0.4W1 +.8W2  +0.3W3 +1.0W4  <= 1
0.9W1 +.4W2  +0.4W3 +0.2W4  <= 1
1.0W1 +.5W2  +1.0W3 +0.3W4  <= 1
0.4W1 +.7W2  +0.7W3 +0.8W4  <= 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU D – OSD-CCR

```
MAX .4 W1 + .8 W2 + .3 W3 + 1. W4

ST

0.5W1 +1W2   +0.3W3 +0.5W4  <= 1
0.7W1 +1W2   +0.3W3 +1.0W4  <= 1
0.8W1 +.7W2  +1.0W3 +0.5W4  <= 1
0.4W1 +.8W2  +0.3W3 +1.0W4  <= 1
0.9W1 +.4W2  +0.4W3 +0.2W4  <= 1
1.0W1 +.5W2  +1.0W3 +0.3W4  <= 1
0.4W1 +.7W2  +0.7W3 +0.8W4  <= 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU E – OSD-CCR

```
MAX .9 W1 + .4 W2 + .4 W3 + .2 W4

ST

0.5W1 +1W2   +0.3W3 +0.5W4  <= 1
```

```
0.7W1 +1W2   +0.3W3 +1.0W4  <= 1
0.8W1 +.7W2  +1.0W3 +0.5W4  <= 1
0.4W1 +.8W2  +0.3W3 +1.0W4  <= 1
0.9W1 +.4W2  +0.4W3 +0.2W4  <= 1
1.0W1 +.5W2  +1.0W3 +0.3W4  <= 1
0.4W1 +.7W2  +0.7W3 +0.8W4  <= 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU F – OSD-CCR

```
MAX 1. W1 + .5 W2 + 1. W3 + .3 W4

ST

0.5W1 +1W2   +0.3W3 +0.5W4  <= 1
0.7W1 +1W2   +0.3W3 +1.0W4  <= 1
0.8W1 +.7W2  +1.0W3 +0.5W4  <= 1
0.4W1 +.8W2  +0.3W3 +1.0W4  <= 1
0.9W1 +.4W2  +0.4W3 +0.2W4  <= 1
1.0W1 +.5W2  +1.0W3 +0.3W4  <= 1
0.4W1 +.7W2  +0.7W3 +0.8W4  <= 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU G – OSD-CCR

```
MAX .4 W1 + .4 W2 + .7 W3 + .8 W4

ST

0.5W1 +1W2   +0.3W3 +0.5W4  <= 1
0.7W1 +1W2   +0.3W3 +1.0W4  <= 1
0.8W1 +.7W2  +1.0W3 +0.5W4  <= 1
0.4W1 +.8W2  +0.3W3 +1.0W4  <= 1
0.9W1 +.4W2  +0.4W3 +0.2W4  <= 1
1.0W1 +.5W2  +1.0W3 +0.3W4  <= 1
0.4W1 +.7W2  +0.7W3 +0.8W4  <= 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

RESULTS DMU A – OSD-CCR

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      1.000000
```

```
 VARIABLE        VALUE          REDUCED COST
       W1         0.000000          0.200000
       W2         1.000000          0.000000
       W3         0.000000          0.000000
       W4         0.000000          0.500000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.000000
       3)         0.000000          1.000000
       4)         0.300000          0.000000
       5)         0.200000          0.000000
       6)         0.600000          0.000000
       7)         0.500000          0.000000
       8)         0.300000          0.000000
       9)         0.000000          0.000000
      10)         1.000000          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000

NO. ITERATIONS=       1
```

RESULTS DMU B – OSD-CCR

```
LP OPTIMUM FOUND AT STEP      1

       OBJECTIVE FUNCTION VALUE

       1)      1.000000

 VARIABLE        VALUE          REDUCED COST
       W1         0.000000          0.000000
       W2         1.000000          0.000000
       W3         0.000000          0.000000
       W4         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.000000
       3)         0.000000          1.000000
       4)         0.300000          0.000000
       5)         0.200000          0.000000
       6)         0.600000          0.000000
       7)         0.500000          0.000000
       8)         0.300000          0.000000
       9)         0.000000          0.000000
      10)         1.000000          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000

NO. ITERATIONS=       1
```

RESULTS DMU C – OSD-CCR

```
LP OPTIMUM FOUND AT STEP      2

       OBJECTIVE FUNCTION VALUE

       1)      1.000000

 VARIABLE        VALUE          REDUCED COST
```

```
       W1         0.000000          0.000000
       W2         0.000000          0.000000
       W3         1.000000          0.000000
       W4         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.700000          0.000000
       3)         0.700000          0.000000
       4)         0.000000          1.000000
       5)         0.700000          0.000000
       6)         0.600000          0.000000
       7)         0.000000          0.000000
       8)         0.300000          0.000000
       9)         0.000000          0.000000
      10)         0.000000          0.000000
      11)         1.000000          0.000000
      12)         0.000000          0.000000

 NO. ITERATIONS=       2
```

RESULTS DMU D – OSD-CCR

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      1.000000

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          0.000000
        W2         0.000000          0.000000
        W3         0.000000          0.000000
        W4         1.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.500000          0.000000
        3)         0.000000          0.000000
        4)         0.500000          0.000000
        5)         0.000000          1.000000
        6)         0.800000          0.000000
        7)         0.700000          0.000000
        8)         0.200000          0.000000
        9)         0.000000          0.000000
       10)         0.000000          0.000000
       11)         0.000000          0.000000
       12)         1.000000          0.000000

 NO. ITERATIONS=       1
```

RESULTS DMU E – OSD-CCR

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)     0.9000000

  VARIABLE        VALUE          REDUCED COST
        W1         1.000000          0.000000
        W2         0.000000          0.050000
```

```
       W3         0.000000          0.500000
       W4         0.000000          0.070000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.500000          0.000000
       3)         0.300000          0.000000
       4)         0.200000          0.000000
       5)         0.600000          0.000000
       6)         0.100000          0.000000
       7)         0.000000          0.900000
       8)         0.600000          0.000000
       9)         1.000000          0.000000
      10)         0.000000          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000

 NO. ITERATIONS=       1
```

RESULTS DMU F – OSD-CCR

```
 LP OPTIMUM FOUND AT STEP      1

       OBJECTIVE FUNCTION VALUE

       1)      1.000000

 VARIABLE        VALUE          REDUCED COST
       W1         1.000000          0.000000
       W2         0.000000          0.000000
       W3         0.000000          0.000000
       W4         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.500000          0.000000
       3)         0.300000          0.000000
       4)         0.200000          0.000000
       5)         0.600000          0.000000
       6)         0.100000          0.000000
       7)         0.000000          1.000000
       8)         0.600000          0.000000
       9)         1.000000          0.000000
      10)         0.000000          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000

 NO. ITERATIONS=       1
```

RESULTS DMU G – OSD-CCR

```
 LP OPTIMUM FOUND AT STEP      2

       OBJECTIVE FUNCTION VALUE

       1)      1.000000

 VARIABLE        VALUE          REDUCED COST
       W1         0.000000          0.000000
       W2         0.000000          0.300000
       W3         0.434783          0.000000
       W4         0.869565          0.000000
```

```
      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.434783          0.000000
       3)         0.000000          0.000000
       4)         0.130435          0.000000
       5)         0.000000          0.000000
       6)         0.652174          0.000000
       7)         0.304348          0.000000
       8)         0.000000          1.000000
       9)         0.000000          0.000000
      10)         0.000000          0.000000
      11)         0.434783          0.000000
      12)         0.869565          0.000000

 NO. ITERATIONS=       2
```

Lindo programs and output for OSD-IP model

DMU A – OSD-IP

```
MIN Z1+Z2+Z3+Z4+Z5+Z6+Z7

ST

0.5 W1 + 1.0 W2    + 0.3 W3 + 0.5 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 -
Z1 <= 0
0.7 W1 + 1.0 W2    + 0.3 W3 + 1.0 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 -
Z2 <= 0
0.8 W1 + 0.7 W2    + 1.0 W3 + 0.5 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 -
Z3 <= 0
0.4 W1 + 0.8 W2    + 0.3 W3 + 1.0 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 -
Z4 <= 0
0.9 W1 + 0.4 W2    + 0.4 W3 + 0.2 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 -
Z5 <= 0
1.0 W1 + 0.5 W2    + 1.0 W3 + 0.3 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 -
Z6 <= 0
0.4 W1 + 0.7 W2    + 0.7 W3 + 0.8 W4 - 0.5 W1 - 1.0 W2 - 0.3 W3 - 0.5 W4 -
Z7 <= 0

W1 + W2 + W3 + W4 = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END

integer Z1
integer Z2
integer Z3
integer Z4
integer Z5
integer Z6
integer Z7
```

DMU B – OSD-IP

```
MIN Z1+Z2+Z3+Z4+Z5+Z6+Z7
```

```
ST

0.5W1 +1W2   +0.3W3 +0.5W4 -0.7W1 -1W2   -0.3W3 -1.0W4 -Z1 <= 0
0.7W1 +1W2   +0.3W3 +1.0W4 -0.7W1 -1W2   -0.3W3 -1.0W4 -Z2 <= 0
0.8W1 +.7W2  +1.0W3 +0.5W4 -0.7W1 -1W2   -0.3W3 -1.0W4 -Z3 <= 0
0.4W1 +.8W2  +0.3W3 +1.0W4 -0.7W1 -1W2   -0.3W3 -1.0W4 -Z4 <= 0
0.9W1 +.4W2  +0.4W3 +0.2W4 -0.7W1 -1W2   -0.3W3 -1.0W4 -Z5 <= 0
1.0W1 +.5W2  +1.0W3 +0.3W4 -0.7W1 -1W2   -0.3W3 -1.0W4 -Z6 <= 0
0.4W1 +.7W2  +0.7W3 +0.8W4 -0.7W1 -1W2   -0.3W3 -1.0W4 -Z7 <= 0

W1 + W2 + W3 + W4 = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END

integer Z1
integer Z2
integer Z3
integer Z4
integer Z5
integer Z6
integer Z7
```

DMU C – OSD-IP

```
MIN Z1+Z2+Z3+Z4+Z5+Z6+Z7

ST

0.5W1 +1W2   +0.3W3 +0.5W4 -0.8W1 -.7W2  -1.0W3 -0.5W4 -Z1 <= 0
0.7W1 +1W2   +0.3W3 +1.0W4 -0.8W1 -.7W2  -1.0W3 -0.5W4 -Z2 <= 0
0.8W1 +.7W2  +1.0W3 +0.5W4 -0.8W1 -.7W2  -1.0W3 -0.5W4 -Z3 <= 0
0.4W1 +.8W2  +0.3W3 +1.0W4 -0.8W1 -.7W2  -1.0W3 -0.5W4 -Z4 <= 0
0.9W1 +.4W2  +0.4W3 +0.2W4 -0.8W1 -.7W2  -1.0W3 -0.5W4 -Z5 <= 0
1.0W1 +.5W2  +1.0W3 +0.3W4 -0.8W1 -.7W2  -1.0W3 -0.5W4 -Z6 <= 0
0.4W1 +.7W2  +0.7W3 +0.8W4 -0.8W1 -.7W2  -1.0W3 -0.5W4 -Z7 <= 0

W1 + W2 + W3 + W4 = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END

integer Z1
integer Z2
integer Z3
integer Z4
integer Z5
integer Z6
integer Z7
```

DMU D – OSD-IP

```
MIN Z1+Z2+Z3+Z4+Z5+Z6+Z7

ST
```

```
0.5W1 +1W2   +0.3W3 +0.5W4 -0.4W1 -.8W2  -0.3W3 -1.0W4 -Z1 <= 0
0.7W1 +1W2   +0.3W3 +1.0W4 -0.4W1 -.8W2  -0.3W3 -1.0W4 -Z2 <= 0
0.8W1 +.7W2  +1.0W3 +0.5W4 -0.4W1 -.8W2  -0.3W3 -1.0W4 -Z3 <= 0
0.4W1 +.8W2  +0.3W3 +1.0W4 -0.4W1 -.8W2  -0.3W3 -1.0W4 -Z4 <= 0
0.9W1 +.4W2  +0.4W3 +0.2W4 -0.4W1 -.8W2  -0.3W3 -1.0W4 -Z5 <= 0
1.0W1 +.5W2  +1.0W3 +0.3W4 -0.4W1 -.8W2  -0.3W3 -1.0W4 -Z6 <= 0
0.4W1 +.7W2  +0.7W3 +0.8W4 -0.4W1 -.8W2  -0.3W3 -1.0W4 -Z7 <= 0

W1 + W2 + W3 + W4 = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END

integer Z1
integer Z2
integer Z3
integer Z4
integer Z5
integer Z6
integer Z7
```

DMU E – OSD-IP

```
MIN Z1+Z2+Z3+Z4+Z5+Z6+Z7

ST

0.5W1 +1W2   +0.3W3 +0.5W4 -0.9W1 -.4W2  -0.4W3 -.2W4 -Z1 <= 0
0.7W1 +1W2   +0.3W3 +1.0W4 -0.9W1 -.4W2  -0.4W3 -.2W4 -Z2 <= 0
0.8W1 +.7W2  +1.0W3 +0.5W4 -0.9W1 -.4W2  -0.4W3 -.2W4 -Z3 <= 0
0.4W1 +.8W2  +0.3W3 +1.0W4 -0.9W1 -.4W2  -0.4W3 -.2W4 -Z4 <= 0
0.9W1 +.4W2  +0.4W3 +0.2W4 -0.9W1 -.4W2  -0.4W3 -.2W4 -Z5 <= 0
1.0W1 +.5W2  +1.0W3 +0.3W4 -0.9W1 -.4W2  -0.4W3 -.2W4 -Z6 <= 0
0.4W1 +.7W2  +0.7W3 +0.8W4 -0.9W1 -.4W2  -0.4W3 -.2W4 -Z7 <= 0

W1 + W2 + W3 + W4 = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END

integer Z1
integer Z2
integer Z3
integer Z4
integer Z5
integer Z6
integer Z7
```

DMU F – OSD-IP

```
MIN Z1+Z2+Z3+Z4+Z5+Z6+Z7

ST

0.5W1 +1W2   +0.3W3 +0.5W4 -1.W1  -.5W2  -1.W3  -.3W4 -Z1 <= 0
```

```
0.7W1 +1W2   +0.3W3 +1.0W4 -1.W1  -.5W2  -1.W3  -.3W4 -Z2 <= 0
0.8W1 +.7W2  +1.0W3 +0.5W4 -1.W1  -.5W2  -1.W3  -.3W4 -Z3 <= 0
0.4W1 +.8W2  +0.3W3 +1.0W4 -1.W1  -.5W2  -1.W3  -.3W4 -Z4 <= 0
0.9W1 +.4W2  +0.4W3 +0.2W4 -1.W1  -.5W2  -1.W3  -.3W4 -Z5 <= 0
1.0W1 +.5W2  +1.0W3 +0.3W4 -1.W1  -.5W2  -1.W3  -.3W4 -Z6 <= 0
0.4W1 +.7W2  +0.7W3 +0.8W4 -1.W1  -.5W2  -1.W3  -.3W4 -Z7 <= 0

W1 + W2 + W3 + W4 = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END

integer Z1
integer Z2
integer Z3
integer Z4
integer Z5
integer Z6
integer Z7
```

DMU G – OSD-IP

```
MIN Z1+Z2+Z3+Z4+Z5+Z6+Z7

ST

0.5W1 +1W2   +0.3W3 +0.5W4 -.4W1  -.7W2  -.7W3  -.8W4 -Z1 <= 0
0.7W1 +1W2   +0.3W3 +1.0W4 -.4W1  -.7W2  -.7W3  -.8W4 -Z2 <= 0
0.8W1 +.7W2  +1.0W3 +0.5W4 -.4W1  -.7W2  -.7W3  -.8W4 -Z3 <= 0
0.4W1 +.8W2  +0.3W3 +1.0W4 -.4W1  -.7W2  -.7W3  -.8W4 -Z4 <= 0
0.9W1 +.4W2  +0.4W3 +0.2W4 -.4W1  -.7W2  -.7W3  -.8W4 -Z5 <= 0
1.0W1 +.5W2  +1.0W3 +0.3W4 -.4W1  -.7W2  -.7W3  -.8W4 -Z6 <= 0
0.4W1 +.7W2  +0.7W3 +0.8W4 -.4W1  -.7W2  -.7W3  -.8W4 -Z7 <= 0

W1 + W2 + W3 + W4 = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END

integer Z1
integer Z2
integer Z3
integer Z4
integer Z5
integer Z6
integer Z7
```

RESULTS DMU A – OSD-IP

```
 LP OPTIMUM FOUND AT STEP      5
 OBJECTIVE VALUE =  0.000000000E+00

 FIX ALL VARS.(    7)  WITH RC >   1.00000

 NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       5
```

```
BOUND ON OPTIMUM: 0.0000000E+00
ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       5

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)     0.0000000E+00

 VARIABLE        VALUE          REDUCED COST
       Z1         0.000000          1.000000
       Z2         0.000000          1.000000
       Z3         0.000000          1.000000
       Z4         0.000000          1.000000
       Z5         0.000000          1.000000
       Z6         0.000000          1.000000
       Z7         0.000000          1.000000
       W1         0.000000          0.000000
       W2         0.700000          0.000000
       W3         0.300000          0.000000
       W4         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.000000
       3)         0.000000          0.000000
       4)         0.000000          0.000000
       5)         0.140000          0.000000
       6)         0.390000          0.000000
       7)         0.140000          0.000000
       8)         0.090000          0.000000
       9)         0.000000          0.000000
      10)         0.000000          0.000000
      11)         0.700000          0.000000
      12)         0.300000          0.000000
      13)         0.000000          0.000000

NO. ITERATIONS=       5
BRANCHES=    0 DETERM.=  1.000E    0
```

RESULTS DMU B – OSD-IP

```
LP OPTIMUM FOUND AT STEP      2
OBJECTIVE VALUE =  0.000000000E+00

FIX ALL VARS.(    7)  WITH RC >   1.00000

NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       2
BOUND ON OPTIMUM: 0.0000000E+00
ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       2

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)     0.0000000E+00

 VARIABLE        VALUE          REDUCED COST
       Z1         0.000000          1.000000
       Z2         0.000000          1.000000
```

```
       Z3          0.000000          1.000000
       Z4          0.000000          1.000000
       Z5          0.000000          1.000000
       Z6          0.000000          1.000000
       Z7          0.000000          1.000000
       W1          0.700000          0.000000
       W2          0.000000          0.000000
       W3          0.000000          0.000000
       W4          0.300000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)          0.290000          0.000000
       3)          0.000000          0.000000
       4)          0.080000          0.000000
       5)          0.210000          0.000000
       6)          0.100000          0.000000
       7)          0.000000          0.000000
       8)          0.270000          0.000000
       9)          0.000000          0.000000
      10)          0.700000          0.000000
      11)          0.000000          0.000000
      12)          0.000000          0.000000
      13)          0.300000          0.000000

 NO. ITERATIONS=       2
 BRANCHES=    0 DETERM.=  1.000E    0
```

RESULTS DMU C – OSD-IP

```
 LP OPTIMUM FOUND AT STEP      2
 OBJECTIVE VALUE =  0.000000000E+00

 FIX ALL VARS.(    7)  WITH RC >   1.00000

 NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       2
 BOUND ON OPTIMUM: 0.0000000E+00
 ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       2

 LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)      0.0000000E+00

  VARIABLE        VALUE          REDUCED COST
        Z1          0.000000          1.000000
        Z2          0.000000          1.000000
        Z3          0.000000          1.000000
        Z4          0.000000          1.000000
        Z5          0.000000          1.000000
        Z6          0.000000          1.000000
        Z7          0.000000          1.000000
        W1          0.700000          0.000000
        W2          0.000000          0.000000
        W3          0.000000          0.000000
        W4          0.300000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)          0.290000          0.000000
```

```
       3)         0.000000          0.000000
       4)         0.080000          0.000000
       5)         0.210000          0.000000
       6)         0.100000          0.000000
       7)         0.000000          0.000000
       8)         0.270000          0.000000
       9)         0.000000          0.000000
      10)         0.700000          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000
      13)         0.300000          0.000000

NO. ITERATIONS=       2
BRANCHES=    0 DETERM.=  1.000E    0
```

RESULTS DMU D – OSD-IP

```
LP OPTIMUM FOUND AT STEP      7
OBJECTIVE VALUE =  0.000000000E+00

FIX ALL VARS.(    6)  WITH RC >   1.00000

NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       7
BOUND ON OPTIMUM: 0.0000000E+00
ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       7

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)     0.0000000E+00

 VARIABLE        VALUE          REDUCED COST
       Z1         0.000000          1.000000
       Z2         0.000000          1.000000
       Z3         0.000000          1.000000
       Z4         0.000000          1.000000
       Z5         0.000000          1.000000
       Z6         0.000000          1.000000
       Z7         0.000000          1.000000
       W1         0.000000          0.000000
       W2         0.000000          0.000000
       W3         0.333333          0.000000
       W4         0.666667          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.333333          0.000000
       3)         0.000000          0.000000
       4)         0.100000          0.000000
       5)         0.000000          0.000000
       6)         0.500000          0.000000
       7)         0.233333          0.000000
       8)         0.000000          0.000000
       9)         0.000000          0.000000
      10)         0.000000          0.000000
      11)         0.000000          0.000000
      12)         0.333333          0.000000
      13)         0.666667          0.000000

NO. ITERATIONS=       7
```

```
BRANCHES=    0 DETERM.=  1.000E    0
```

RESULTS DMU E – OSD-IP

```
LP OPTIMUM FOUND AT STEP      2
OBJECTIVE VALUE =  0.100000024

FIX ALL VARS.(    6)  WITH RC >   1.00000
SET       Z6 TO >=     1 AT    1, BND=  -1.000     TWIN=-0.1000E+31     12

NEW INTEGER SOLUTION OF    1.00000000     AT BRANCH      1 PIVOT      12
BOUND ON OPTIMUM:  1.000000
DELETE       Z6 AT LEVEL     1
ENUMERATION COMPLETE. BRANCHES=     1 PIVOTS=      12

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)      1.000000

 VARIABLE        VALUE          REDUCED COST
       Z1         0.000000          1.000000
       Z2         0.000000          1.000000
       Z3         0.000000          1.000000
       Z4         0.000000          1.000000
       Z5         0.000000          1.000000
       Z6         1.000000          1.000000
       Z7         0.000000          1.000000
       W1         0.857143          0.000000
       W2         0.000000          0.000000
       W3         0.142857          0.000000
       W4         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.357143          0.000000
       3)         0.185714          0.000000
       4)         0.000000          0.000000
       5)         0.442857          0.000000
       6)         0.000000          0.000000
       7)         0.828571          0.000000
       8)         0.385714          0.000000
       9)         0.000000          0.000000
      10)         0.857143          0.000000
      11)         0.000000          0.000000
      12)         0.142857          0.000000
      13)         0.000000          0.000000

NO. ITERATIONS=      12
BRANCHES=    1 DETERM.=  1.000E    0
```

RESULTS DMU F – OSD-IP

```
LP OPTIMUM FOUND AT STEP      1
OBJECTIVE VALUE =  0.000000000E+00

FIX ALL VARS.(    7)  WITH RC >   1.00000

NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       1
BOUND ON OPTIMUM: 0.0000000E+00
```

```
ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       1

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)     0.0000000E+00

 VARIABLE        VALUE          REDUCED COST
       Z1         0.000000          1.000000
       Z2         0.000000          1.000000
       Z3         0.000000          1.000000
       Z4         0.000000          1.000000
       Z5         0.000000          1.000000
       Z6         0.000000          1.000000
       Z7         0.000000          1.000000
       W1         1.000000          0.000000
       W2         0.000000          0.000000
       W3         0.000000          0.000000
       W4         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.500000          0.000000
       3)         0.300000          0.000000
       4)         0.200000          0.000000
       5)         0.600000          0.000000
       6)         0.100000          0.000000
       7)         0.000000          0.000000
       8)         0.600000          0.000000
       9)         0.000000          0.000000
      10)         1.000000          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000
      13)         0.000000          0.000000

NO. ITERATIONS=       1
BRANCHES=    0 DETERM.=  1.000E    0
```

RESULTS DMU G – OSD-IP

```
LP OPTIMUM FOUND AT STEP      6
OBJECTIVE VALUE =  0.000000000E+00

FIX ALL VARS.(    7)  WITH RC >   1.00000

NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       6
BOUND ON OPTIMUM: 0.0000000E+00
ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       6

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)     0.0000000E+00

 VARIABLE        VALUE          REDUCED COST
       Z1         0.000000          1.000000
       Z2         0.000000          1.000000
       Z3         0.000000          1.000000
```

```
        Z4          0.000000          1.000000
        Z5          0.000000          1.000000
        Z6          0.000000          1.000000
        Z7          0.000000          1.000000
        W1          0.000000          0.000000
        W2          0.000000          0.000000
        W3          0.333333          0.000000
        W4          0.666667          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)          0.333333          0.000000
        3)          0.000000          0.000000
        4)          0.100000          0.000000
        5)          0.000000          0.000000
        6)          0.500000          0.000000
        7)          0.233333          0.000000
        8)          0.000000          0.000000
        9)          0.000000          0.000000
       10)          0.000000          0.000000
       11)          0.000000          0.000000
       12)          0.333333          0.000000
       13)          0.666667          0.000000

 NO. ITERATIONS=       6
 BRANCHES=    0 DETERM.=  1.000E    0
```

Lindo programs and output for OSD-DA model

DMU A – OSD-DA

```
MIN 0.5 W1 + 1.0 W2 + 0.3 W3 +0.5 W4 + 0.7 W1 + 1.0 W2 +0.3 W3 + 1.0 W4 +
0.8 W1 + 0.7 W2 + 1.0 W3 + 0.5 W4 + 0.4 W1 + 0.8 W2 + 0.3 W3 +1.0 W4 + 0.9
W1    + 0.4 W2 +0.4W3 +0.2 W4 + 1.0 W1 + 0.5 W2 + 1.0 W3 +0.3 W4 + 0.4 W1 +
0.7 W2 +0.7 W3 + 0.8 W4

ST

0.5 W1 + 1.0 W2    + 0.3 W3 + 0.5 W4 = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU B – OSD-DA

```
MIN 0.5W1 +1W2 +0.3W3 +0.5W4 + 0.7W1 +1W2 +0.3W3 +1.0W4 + 0.8W1   +.7W2
+1.0W3 +0.5W4 + 0.4W1 +.8W2 +0.3W3 +1.0W4 + 0.9W1    +.4W2  +0.4W3  +0.2W4  +
1.0W1 +.5W2 +1.0W3 +0.3W4 + 0.4W1      +.7W2 +0.7W3 +0.8W4

ST

! 0.5W1      +1W2   +0.3W3 +0.5W4  = 1
 0.7W1       +1W2   +0.3W3 +1.0W4  = 1
! 0.8W1      +.7W2  +1.0W3 +0.5W4  = 1
! 0.4W1      +.8W2  +0.3W3 +1.0W4  = 1
```

```
! 0.9W1      +.4W2  +0.4W3 +0.2W4  = 1
! 1.0W1      +.5W2  +1.0W3 +0.3W4  = 1
! 0.4W1      +.7W2  +0.7W3 +0.8W4  = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU C – OSD-DA

```
MIN 0.5W1 +1W2 +0.3W3 +0.5W4 + 0.7W1 +1W2 +0.3W3 +1.0W4 + 0.8W1   +.7W2
+1.0W3 +0.5W4 + 0.4W1 +.8W2 +0.3W3 +1.0W4 + 0.9W1    +.4W2  +0.4W3  +0.2W4  +
1.0W1 +.5W2 +1.0W3 +0.3W4 + 0.4W1      +.7W2 +0.7W3 +0.8W4

ST

! 0.5W1      +1W2   +0.3W3 +0.5W4  = 1
! 0.7W1      +1W2   +0.3W3 +1.0W4  = 1
 0.8W1       +.7W2  +1.0W3 +0.5W4  = 1
! 0.4W1      +.8W2  +0.3W3 +1.0W4  = 1
! 0.9W1      +.4W2  +0.4W3 +0.2W4  = 1
! 1.0W1      +.5W2  +1.0W3 +0.3W4  = 1
! 0.4W1      +.7W2  +0.7W3 +0.8W4  = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU D – OSD-DA

```
MIN 0.5W1 +1W2 +0.3W3 +0.5W4 + 0.7W1 +1W2 +0.3W3 +1.0W4 + 0.8W1   +.7W2
+1.0W3 +0.5W4 + 0.4W1 +.8W2 +0.3W3 +1.0W4 + 0.9W1    +.4W2  +0.4W3  +0.2W4  +
1.0W1 +.5W2 +1.0W3 +0.3W4 + 0.4W1      +.7W2 +0.7W3 +0.8W4

ST

! 0.5W1      +1W2   +0.3W3 +0.5W4  = 1
! 0.7W1      +1W2   +0.3W3 +1.0W4  = 1
! 0.8W1      +.7W2  +1.0W3 +0.5W4  = 1
 0.4W1       +.8W2  +0.3W3 +1.0W4  = 1
! 0.9W1      +.4W2  +0.4W3 +0.2W4  = 1
! 1.0W1      +.5W2  +1.0W3 +0.3W4  = 1
! 0.4W1      +.7W2  +0.7W3 +0.8W4  = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU E – OSD-DA

```
MIN 0.5W1 +1W2 +0.3W3 +0.5W4 + 0.7W1 +1W2 +0.3W3 +1.0W4 + 0.8W1   +.7W2
+1.0W3 +0.5W4 + 0.4W1 +.8W2 +0.3W3 +1.0W4 + 0.9W1    +.4W2  +0.4W3  +0.2W4  +
1.0W1 +.5W2 +1.0W3 +0.3W4 + 0.4W1      +.7W2 +0.7W3 +0.8W4

ST
```

```
! 0.5W1      +1W2   +0.3W3 +0.5W4  = 1
! 0.7W1      +1W2   +0.3W3 +1.0W4  = 1
! 0.8W1      +.7W2  +1.0W3 +0.5W4  = 1
! 0.4W1      +.8W2  +0.3W3 +1.0W4  = 1
 0.9W1       +.4W2  +0.4W3 +0.2W4  = 1
! 1.0W1      +.5W2  +1.0W3 +0.3W4  = 1
! 0.4W1      +.7W2  +0.7W3 +0.8W4  = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU F – OSD-DA

```
MIN 0.5W1 +1W2 +0.3W3 +0.5W4 + 0.7W1 +1W2 +0.3W3 +1.0W4 + 0.8W1   +.7W2
+1.0W3 +0.5W4 + 0.4W1 +.8W2 +0.3W3 +1.0W4 + 0.9W1    +.4W2  +0.4W3  +0.2W4  +
1.0W1 +.5W2 +1.0W3 +0.3W4 + 0.4W1      +.7W2 +0.7W3 +0.8W4

ST

! 0.5W1      +1W2   +0.3W3 +0.5W4  = 1
! 0.7W1      +1W2   +0.3W3 +1.0W4  = 1
! 0.8W1      +.7W2  +1.0W3 +0.5W4  = 1
! 0.4W1      +.8W2  +0.3W3 +1.0W4  = 1
! 0.9W1      +.4W2  +0.4W3 +0.2W4  = 1
 1.0W1       +.5W2  +1.0W3 +0.3W4  = 1
! 0.4W1      +.7W2  +0.7W3 +0.8W4  = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU G – OSD-DA

```
MIN 0.5W1 +1W2 +0.3W3 +0.5W4 + 0.7W1 +1W2 +0.3W3 +1.0W4 + 0.8W1   +.7W2
+1.0W3 +0.5W4 + 0.4W1 +.8W2 +0.3W3 +1.0W4 + 0.9W1    +.4W2  +0.4W3  +0.2W4  +
1.0W1 +.5W2 +1.0W3 +0.3W4 + 0.4W1      +.7W2 +0.7W3 +0.8W4

ST

! 0.5W1      +1W2   +0.3W3 +0.5W4  = 1
! 0.7W1      +1W2   +0.3W3 +1.0W4  = 1
! 0.8W1      +.7W2  +1.0W3 +0.5W4  = 1
! 0.4W1      +.8W2  +0.3W3 +1.0W4  = 1
! 0.9W1      +.4W2  +0.4W3 +0.2W4  = 1
! 1.0W1      +.5W2  +1.0W3 +0.3W4  = 1
 0.4W1       +.7W2  +0.7W3 +0.8W4  = 1

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

RESULTS DMU A – OSD-DA

```
 LP OPTIMUM FOUND AT STEP      1
```

```
       OBJECTIVE FUNCTION VALUE

       1)      5.100000

 VARIABLE        VALUE          REDUCED COST
       W1         0.000000          2.150000
       W2         1.000000          0.000000
       W3         0.000000          2.470000
       W4         0.000000          1.750000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000         -5.100000
       3)         0.000000          0.000000
       4)         1.000000          0.000000
       5)         0.000000          0.000000
       6)         0.000000          0.000000

 NO. ITERATIONS=       1
```

RESULTS DMU B – OSD-DA

```
 LP OPTIMUM FOUND AT STEP      1

       OBJECTIVE FUNCTION VALUE

       1)      4.300000

 VARIABLE        VALUE          REDUCED COST
       W1         0.000000          1.690000
       W2         0.000000          0.800000
       W3         0.000000          2.710000
       W4         1.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000         -4.300000
       3)         0.000000          0.000000
       4)         0.000000          0.000000
       5)         0.000000          0.000000
       6)         1.000000          0.000000

 NO. ITERATIONS=       1
```

RESULTS DMU C – OSD-DA

```
 LP OPTIMUM FOUND AT STEP      1

       OBJECTIVE FUNCTION VALUE

       1)      4.000000

 VARIABLE        VALUE          REDUCED COST
       W1         0.000000          1.500000
       W2         0.000000          2.300000
       W3         1.000000          0.000000
       W4         0.000000          2.300000
```

```
      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000        -4.000000
       3)         0.000000         0.000000
       4)         0.000000         0.000000
       5)         1.000000         0.000000
       6)         0.000000         0.000000

 NO. ITERATIONS=       1
```

RESULTS DMU D – OSD-DA

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      4.300000

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          2.980000
        W2         0.000000          1.660000
        W3         0.000000          2.710000
        W4         1.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000        -4.300000
       3)         0.000000         0.000000
       4)         0.000000         0.000000
       5)         0.000000         0.000000
       6)         1.000000         0.000000

 NO. ITERATIONS=       1
```

RESULTS DMU E – OSD-DA

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      5.222222

  VARIABLE        VALUE          REDUCED COST
        W1         1.111111          0.000000
        W2         0.000000          3.011111
        W3         0.000000          1.911111
        W4         0.000000          3.255556


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000        -5.222222
       3)         1.111111         0.000000
       4)         0.000000         0.000000
       5)         0.000000         0.000000
       6)         0.000000         0.000000

 NO. ITERATIONS=       1
```

RESULTS DMU F – OSD-DA

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      4.000000

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          0.700000
        W2         0.000000          3.100000
        W3         1.000000          0.000000
        W4         0.000000          3.100000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -4.000000
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)         1.000000          0.000000
        6)         0.000000          0.000000

 NO. ITERATIONS=       1
```

RESULTS DMU G – OSD-DA

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      5.375000

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          2.550000
        W2         0.000000          1.337500
        W3         0.000000          0.237500
        W4         1.250000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -5.375000
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)         0.000000          0.000000
        6)         1.250000          0.000000

 NO. ITERATIONS=       1
```

Lindo programs and output for CCR-AR model

DMU A – CCR-AR

```
MAX 0.5 W1 + 1.0 W2 + 0.3 W3 + 0.5 W4

ST

0.5 W1 + 1.0 W2 + 0.3 W3 + 0.5 W4 <= 1
0.7 W1 + 1.0 W2 + 0.3 W3 + 1.0 W4 <= 1
0.8 W1 + 0.7 W2 + 1.0 W3 + 0.5 W4 <= 1
0.4 W1 + 0.8 W2 + 0.3 W3 + 1.0 W4 <= 1
```

```
0.9 W1 + 0.4 W2 + 0.4 W3 + 0.2 W4 <= 1
1.0 W1 + 0.5 W2 + 1.0 W3 + 0.3 W4 <= 1
0.4 W1 + 0.7 W2 + 0.7 W3 + 0.8 W4 <= 1

 0.75 W1 - 1.00 W2 <= 0
-2.00 W1 + 1.00 W2 <= 0

 0.74 W1 - 1.00 W3 <= 0
-1.00 W1 + 1.00 W3 <= 0

 1.50 W1 - 1.00 W4 <= 0
-2.00 W1 + 1.00 W4 <= 0

 0.50 W2 - 1.00 W3 <= 0
-1.00 W2 + 1.00 W3 <= 0

 1.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

 2.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU B – CCR-AR

```
MAX .7 W1 + 1. W2 + .3 W3 + 1. W4

ST

0.5W1 +1W2   +0.3W3 +0.5W4  <= 1
0.7W1 +1W2   +0.3W3 +1.0W4  <= 1
0.8W1 +.7W2  +1.0W3 +0.5W4  <= 1
0.4W1 +.8W2  +0.3W3 +1.0W4  <= 1
0.9W1 +.4W2  +0.4W3 +0.2W4  <= 1
1.0W1 +.5W2  +1.0W3 +0.3W4  <= 1
0.4W1 +.7W2  +0.7W3 +0.8W4  <= 1

 0.75 W1 - 1.00 W2 <= 0
-2.00 W1 + 1.00 W2 <= 0

 0.74 W1 - 1.00 W3 <= 0
-1.00 W1 + 1.00 W3 <= 0

 1.50 W1 - 1.00 W4 <= 0
-2.00 W1 + 1.00 W4 <= 0

 0.50 W2 - 1.00 W3 <= 0
-1.00 W2 + 1.00 W3 <= 0

 1.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

 2.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

W1 > 0
W2 > 0
```

```
W3 > 0
W4 > 0

END
```

DMU C – CCR-AR

```
MAX .8 W1 + .7 W2 + 1. W3 + .5 W4

ST

0.5W1 +1W2   +0.3W3 +0.5W4  <= 1
0.7W1 +1W2   +0.3W3 +1.0W4  <= 1
0.8W1 +.7W2  +1.0W3 +0.5W4  <= 1
0.4W1 +.8W2  +0.3W3 +1.0W4  <= 1
0.9W1 +.4W2  +0.4W3 +0.2W4  <= 1
1.0W1 +.5W2  +1.0W3 +0.3W4  <= 1
0.4W1 +.7W2  +0.7W3 +0.8W4  <= 1

 0.75 W1 - 1.00 W2 <= 0
-2.00 W1 + 1.00 W2 <= 0

 0.74 W1 - 1.00 W3 <= 0
-1.00 W1 + 1.00 W3 <= 0

 1.50 W1 - 1.00 W4 <= 0
-2.00 W1 + 1.00 W4 <= 0

 0.50 W2 - 1.00 W3 <= 0
-1.00 W2 + 1.00 W3 <= 0

 1.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

 2.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU D – CCR-AR

```
MAX .4 W1 + .8 W2 + .3 W3 + 1. W4

ST

0.5W1 +1W2   +0.3W3 +0.5W4  <= 1
0.7W1 +1W2   +0.3W3 +1.0W4  <= 1
0.8W1 +.7W2  +1.0W3 +0.5W4  <= 1
0.4W1 +.8W2  +0.3W3 +1.0W4  <= 1
0.9W1 +.4W2  +0.4W3 +0.2W4  <= 1
1.0W1 +.5W2  +1.0W3 +0.3W4  <= 1
0.4W1 +.7W2  +0.7W3 +0.8W4  <= 1

 0.75 W1 - 1.00 W2 <= 0
-2.00 W1 + 1.00 W2 <= 0

 0.74 W1 - 1.00 W3 <= 0
-1.00 W1 + 1.00 W3 <= 0
```

```
 1.50 W1 - 1.00 W4 <= 0
-2.00 W1 + 1.00 W4 <= 0

 0.50 W2 - 1.00 W3 <= 0
-1.00 W2 + 1.00 W3 <= 0

 1.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

 2.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU E – CCR-AR

```
MAX .9 W1 + .4 W2 + .4 W3 + .2 W4

ST

0.5W1 +1W2   +0.3W3 +0.5W4  <= 1
0.7W1 +1W2   +0.3W3 +1.0W4  <= 1
0.8W1 +.7W2  +1.0W3 +0.5W4  <= 1
0.4W1 +.8W2  +0.3W3 +1.0W4  <= 1
0.9W1 +.4W2  +0.4W3 +0.2W4  <= 1
1.0W1 +.5W2  +1.0W3 +0.3W4  <= 1
0.4W1 +.7W2  +0.7W3 +0.8W4  <= 1

 0.75 W1 - 1.00 W2 <= 0
-2.00 W1 + 1.00 W2 <= 0

 0.74 W1 - 1.00 W3 <= 0
-1.00 W1 + 1.00 W3 <= 0

 1.50 W1 - 1.00 W4 <= 0
-2.00 W1 + 1.00 W4 <= 0

 0.50 W2 - 1.00 W3 <= 0
-1.00 W2 + 1.00 W3 <= 0

 1.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

 2.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU F – CCR-AR

```
MAX 1. W1 + .5 W2 + 1. W3 + .3 W4

ST
```

```
0.5W1 +1W2   +0.3W3 +0.5W4  <= 1
0.7W1 +1W2   +0.3W3 +1.0W4  <= 1
0.8W1 +.7W2  +1.0W3 +0.5W4  <= 1
0.4W1 +.8W2  +0.3W3 +1.0W4  <= 1
0.9W1 +.4W2  +0.4W3 +0.2W4  <= 1
1.0W1 +.5W2  +1.0W3 +0.3W4  <= 1
0.4W1 +.7W2  +0.7W3 +0.8W4  <= 1

 0.75 W1 - 1.00 W2 <= 0
-2.00 W1 + 1.00 W2 <= 0

 0.74 W1 - 1.00 W3 <= 0
-1.00 W1 + 1.00 W3 <= 0

 1.50 W1 - 1.00 W4 <= 0
-2.00 W1 + 1.00 W4 <= 0

 0.50 W2 - 1.00 W3 <= 0
-1.00 W2 + 1.00 W3 <= 0

 1.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

 2.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

DMU G – CCR-AR

```
MAX .4 W1 + .4 W2 + .7 W3 + .8 W4

ST

0.5W1 +1W2   +0.3W3 +0.5W4  <= 1
0.7W1 +1W2   +0.3W3 +1.0W4  <= 1
0.8W1 +.7W2  +1.0W3 +0.5W4  <= 1
0.4W1 +.8W2  +0.3W3 +1.0W4  <= 1
0.9W1 +.4W2  +0.4W3 +0.2W4  <= 1
1.0W1 +.5W2  +1.0W3 +0.3W4  <= 1
0.4W1 +.7W2  +0.7W3 +0.8W4  <= 1

 0.75 W1 - 1.00 W2 <= 0
-2.00 W1 + 1.00 W2 <= 0

 0.74 W1 - 1.00 W3 <= 0
-1.00 W1 + 1.00 W3 <= 0

 1.50 W1 - 1.00 W4 <= 0
-2.00 W1 + 1.00 W4 <= 0

 0.50 W2 - 1.00 W3 <= 0
-1.00 W2 + 1.00 W3 <= 0

 1.00 W2 - 1.00 W4 <= 0
-2.00 W2 + 1.00 W4 <= 0

 2.00 W2 - 1.00 W4 <= 0
```

```
-2.00 W2 + 1.00 W4 <= 0

W1 > 0
W2 > 0
W3 > 0
W4 > 0

END
```

RESULTS DMU A – CCR-AR

```
 LP OPTIMUM FOUND AT STEP      6

        OBJECTIVE FUNCTION VALUE

        1)      0.7007874

  VARIABLE        VALUE          REDUCED COST
        W1         0.314961          0.000000
        W2         0.236220          0.000000
        W3         0.236220          0.000000
        W4         0.472441          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.299213          0.000000
        3)         0.000000          0.700787
        4)         0.110236          0.000000
        5)         0.141732          0.000000
        6)         0.433071          0.000000
        7)         0.188976          0.000000
        8)         0.165354          0.000000
        9)         0.000000          0.000000
       10)         0.393701          0.000000
       11)         0.003150          0.000000
       12)         0.078740          0.000000
       13)         0.000000          0.006299
       14)         0.157480          0.000000
       15)         0.118110          0.000000
       16)         0.000000          0.089764
       17)         0.236220          0.000000
       18)         0.000000          0.000000
       19)         0.000000          0.194488
       20)         0.000000          0.000000
       21)         0.314961          0.000000
       22)         0.236220          0.000000
       23)         0.236220          0.000000
       24)         0.472441          0.000000

 NO. ITERATIONS=       6
```

RESULTS DMU B – CCR-AR

```
 LP OPTIMUM FOUND AT STEP      4

        OBJECTIVE FUNCTION VALUE

        1)      1.000000

  VARIABLE        VALUE          REDUCED COST
        W1         0.254972          0.000000
```

W2	0.254972	0.000000
W3	0.188679	0.000000
W4	0.509944	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.305966	0.000000
3)	0.000000	1.000000
4)	0.173891	0.000000
5)	0.127486	0.000000
6)	0.491076	0.000000
7)	0.275880	0.000000
8)	0.179500	0.000000
9)	0.063743	0.000000
10)	0.254972	0.000000
11)	0.000000	0.000000
12)	0.066293	0.000000
13)	0.127486	0.000000
14)	0.000000	0.000000
15)	0.061193	0.000000
16)	0.066293	0.000000
17)	0.254972	0.000000
18)	0.000000	0.000000
19)	0.000000	0.000000
20)	0.000000	0.000000
21)	0.254972	0.000000
22)	0.254972	0.000000
23)	0.188679	0.000000
24)	0.509944	0.000000

NO. ITERATIONS= 4

RESULTS DMU C – CCR-AR

LP OPTIMUM FOUND AT STEP 5

OBJECTIVE FUNCTION VALUE

1) 0.8897638

VARIABLE	VALUE	REDUCED COST
W1	0.314961	0.000000
W2	0.236220	0.000000
W3	0.236220	0.000000
W4	0.472441	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.299213	0.000000
3)	0.000000	0.889764
4)	0.110236	0.000000
5)	0.141732	0.000000
6)	0.433071	0.000000
7)	0.188976	0.000000
8)	0.165354	0.000000
9)	0.000000	0.000000
10)	0.393701	0.000000
11)	0.003150	0.000000
12)	0.078740	0.000000
13)	0.000000	0.118110
14)	0.157480	0.000000

```
     15)          0.118110          0.000000
     16)          0.000000          0.733071
     17)          0.236220          0.000000
     18)          0.000000          0.000000
     19)          0.000000          0.271654
     20)          0.000000          0.000000
     21)          0.314961          0.000000
     22)          0.236220          0.000000
     23)          0.236220          0.000000
     24)          0.472441          0.000000

NO. ITERATIONS=       5
```

RESULTS DMU D – CCR-AR

```
LP OPTIMUM FOUND AT STEP      5

       OBJECTIVE FUNCTION VALUE

       1)      0.8750000

 VARIABLE        VALUE          REDUCED COST
       W1         0.250000          0.000000
       W2         0.250000          0.000000
       W3         0.250000          0.000000
       W4         0.500000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.300000          0.000000
       3)         0.000000          0.875000
       4)         0.125000          0.000000
       5)         0.125000          0.000000
       6)         0.475000          0.000000
       7)         0.225000          0.000000
       8)         0.150000          0.000000
       9)         0.062500          0.000000
      10)         0.250000          0.000000
      11)         0.065000          0.000000
      12)         0.000000          0.000000
      13)         0.125000          0.000000
      14)         0.000000          0.106250
      15)         0.125000          0.000000
      16)         0.000000          0.037500
      17)         0.250000          0.000000
      18)         0.000000          0.000000
      19)         0.000000          0.000000
      20)         0.000000          0.018750
      21)         0.250000          0.000000
      22)         0.250000          0.000000
      23)         0.250000          0.000000
      24)         0.500000          0.000000

NO. ITERATIONS=       5
```

RESULTS DMU E – CCR-AR

```
LP OPTIMUM FOUND AT STEP      6

       OBJECTIVE FUNCTION VALUE
```

```
       1)      0.5669292

 VARIABLE        VALUE          REDUCED COST
       W1         0.314961          0.000000
       W2         0.236220          0.000000
       W3         0.236220          0.000000
       W4         0.472441          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.299213          0.000000
       3)         0.000000          0.566929
       4)         0.110236          0.000000
       5)         0.141732          0.000000
       6)         0.433071          0.000000
       7)         0.188976          0.000000
       8)         0.165354          0.000000
       9)         0.000000          0.000000
      10)         0.393701          0.000000
      11)         0.003150          0.000000
      12)         0.078740          0.000000
      13)         0.000000          0.335433
      14)         0.157480          0.000000
      15)         0.118110          0.000000
      16)         0.000000          0.229921
      17)         0.236220          0.000000
      18)         0.000000          0.000000
      19)         0.000000          0.031496
      20)         0.000000          0.000000
      21)         0.314961          0.000000
      22)         0.236220          0.000000
      23)         0.236220          0.000000
      24)         0.472441          0.000000

 NO. ITERATIONS=       6
```

RESULTS DMU F – CCR-AR

```
 LP OPTIMUM FOUND AT STEP      4

        OBJECTIVE FUNCTION VALUE

       1)      0.8110236

 VARIABLE        VALUE          REDUCED COST
       W1         0.314961          0.000000
       W2         0.236220          0.000000
       W3         0.236220          0.000000
       W4         0.472441          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.299213          0.000000
       3)         0.000000          0.811024
       4)         0.110236          0.000000
       5)         0.141732          0.000000
       6)         0.433071          0.000000
       7)         0.188976          0.000000
       8)         0.165354          0.000000
       9)         0.000000          0.000000
```

```
      10)         0.393701          0.000000
      11)         0.003150          0.000000
      12)         0.078740          0.000000
      13)         0.000000          0.288189
      14)         0.157480          0.000000
      15)         0.118110          0.000000
      16)         0.000000          0.756693
      17)         0.236220          0.000000
      18)         0.000000          0.000000
      19)         0.000000          0.222835
      20)         0.000000          0.000000
      21)         0.314961          0.000000
      22)         0.236220          0.000000
      23)         0.236220          0.000000
      24)         0.472441          0.000000

 NO. ITERATIONS=       4
```

RESULTS DMU G – CCR-AR

```
 LP OPTIMUM FOUND AT STEP      8

       OBJECTIVE FUNCTION VALUE

       1)      0.7750000

  VARIABLE        VALUE          REDUCED COST
       W1         0.250000          0.000000
       W2         0.250000          0.000000
       W3         0.250000          0.000000
       W4         0.500000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.300000          0.000000
       3)         0.000000          0.775000
       4)         0.125000          0.000000
       5)         0.125000          0.000000
       6)         0.475000          0.000000
       7)         0.225000          0.000000
       8)         0.150000          0.000000
       9)         0.062500          0.000000
      10)         0.250000          0.000000
      11)         0.065000          0.000000
      12)         0.000000          0.000000
      13)         0.125000          0.000000
      14)         0.000000          0.071250
      15)         0.125000          0.000000
      16)         0.000000          0.467500
      17)         0.250000          0.000000
      18)         0.000000          0.000000
      19)         0.000000          0.046250
      20)         0.000000          0.000000
      21)         0.250000          0.000000
      22)         0.250000          0.000000
      23)         0.250000          0.000000
      24)         0.500000          0.000000

 NO. ITERATIONS=       8
```

Capital City Site Selection [TONE00]

Seven Steps using Expert Elicited Weights

Step 1 - Collect & Orient Measures

Location	C1	C2	C3	C4
A	5	10	3	5
B	7	10	3	10
C	8	7	10	5
D	4	8	3	10
E	9	4	4	2
F	10	5	10	3
G	4	7	7	8

Step 2 - Level Scales

Alternative	C1	C2	C3	C4	Total
A	0.500	1.000	0.300	0.500	2.300
B	0.700	1.000	0.300	1.000	3.000
C	0.800	0.700	1.000	0.500	3.000
D	0.400	0.800	0.300	1.000	2.500
E	0.900	0.400	0.400	0.200	1.900
F	1.000	0.500	1.000	0.300	2.800
G	0.400	0.700	0.700	0.800	2.600

Step 3 - Choose Weights

Expert Elicited Weights Using AHP

Evaluator	W1	W2	W3	W4
Expert 1	1.670	3.330	1.670	3.330
Expert 2	2.110	3.160	1.580	3.160
Expert 3	2.500	1.880	1.880	3.750
Expert 4	2.000	2.000	2.000	4.000
Expert 5	2.400	1.900	1.900	3.800

Expert Elicited Weights Normalized

Evaluator	W1	W2	W3	W4
Expert 1	0.167	0.333	0.167	0.333
Expert 2	0.211	0.316	0.158	0.316
Expert 3	0.250	0.188	0.188	0.375
Expert 4	0.200	0.200	0.200	0.400
Expert 5	0.240	0.190	0.190	0.380

Step 4 - Stack Weights

Expert STACKED WEIGHTS

W1	W2	W3	W4
1.068	1.226	0.903	1.803

Step 5 - Calculate Weight Importance Factor

Expert WIF

W1	W2	W3	W4
21%	25%	18%	36%

Step 6 - Calculate Scores

Expert SCORES

Alternative	Expert-Score
A	0.587
B	0.810
C	0.703
D	0.696
E	0.435
F	0.625
G	0.672

Step 7 - Rank Alternatives

Expert-RANKS

Alternative	Expert-Rank
A	6
B	1
C	2
D	3
E	7
F	5
G	4

Capital City Site Selection [TONE00]

Seven Steps using OSD-CCR

Step 1 - Collect & Orient Measures

Location	C1	C2	C3	C4
A	5	10	3	5
B	7	10	3	10
C	8	7	10	5
D	4	8	3	10
E	9	4	4	2
F	10	5	10	3
G	4	7	7	8

Step 2 - Level Scales

Alternative	C1	C2	C3	C4	Total
A	0.500	1.000	0.300	0.500	2.300
B	0.700	1.000	0.300	1.000	3.000
C	0.800	0.700	1.000	0.500	3.000
D	0.400	0.800	0.300	1.000	2.500
E	0.900	0.400	0.400	0.200	1.900
F	1.000	0.500	1.000	0.300	2.800
G	0.400	0.700	0.700	0.800	2.600

Step 3 - Choose Weights

OSD-CCR RAW RESULTS

Alternative	W1	W2	W3	W4	OSD-CCR SC
A	0.000	1.000	0.000	0.000	1.000
B	0.000	1.000	0.000	0.000	1.000
C	0.000	0.886	0.380	0.000	1.000
D	0.000	0.000	0.000	1.000	1.000
E	1.000	0.000	0.000	0.000	0.900
F	1.000	0.000	0.000	0.000	1.000
G	0.175	0.000	0.496	0.729	1.000

OSD-CCR NORM. RESULTS

Alternative	W1	W2	W3	W4
A	0.000	1.000	0.000	0.000
B	0.000	1.000	0.000	0.000
C	0.000	0.700	0.300	0.000
D	0.000	0.000	0.000	1.000
E	1.000	0.000	0.000	0.000
F	1.000	0.000	0.000	0.000
G	0.125	0.000	0.354	0.521

Step 4 - Stack Weights

OSD-CCR STACKED WEIGHTS

W1	W2	W3	W4
2.125	2.700	0.654	1.521

Step 5 - Calculate Weight Importance Factor

OSD-CCR WIF

W1	W2	W3	W4
30%	39%	9%	22%

Step 6 - Calculate Scores

OSD-CCR C-SCORES

Alternative	C-Score
A	0.674
B	0.844
C	0.715
D	0.675
E	0.508
F	0.655
G	0.631

Step 7 - Rank Alternatives

OSD-CCR C-RANKS

Alternative	C-Rank
A	4
B	1
C	2
D	3
E	7
F	5
G	6

Capital City Site Selection [TONE00]

Seven Steps using OSD-IP

Step 1 - Collect & Orient Measures

Location	C1	C2	C3	C4
A	5	10	3	5
B	7	10	3	10
C	8	7	10	5
D	4	8	3	10
E	9	4	4	2
F	10	5	10	3
G	4	7	7	8

Step 2 - Level Scales

Alternative	C1	C2	C3	C4	Total
A	0.50	1.00	0.30	0.50	2.30
B	0.70	1.00	0.30	1.00	3.00
C	0.80	0.70	1.00	0.50	3.00
D	0.40	0.80	0.30	1.00	2.50
E	0.90	0.40	0.40	0.20	1.90
F	1.00	0.50	1.00	0.30	2.80
G	0.40	0.70	0.70	0.80	2.60

Step 3 - Choose Weights

OSD-IP RAW RESULTS

Alternative	W1	W2	W3	W4	No. Above	Max Rank
A		0.700	0.300		0	1
B	0.700			0.300	0	1
C			1.000		0	1
D			0.333	0.667	0	1
E	0.857		0.143		1	2
F	1.000				0	1
G			0.333	0.667	0	1

OSD-IP NORM. RESULTS

Alternative	W1	W2	W3	W4
A	0.000	0.700	0.300	0.000
B	0.700	0.000	0.000	0.300
C	0.000	0.000	1.000	0.000
D	0.000	0.000	0.333	0.667
E	0.857	0.000	0.143	0.000
F	1.000	0.000	0.000	0.000
G	0.000	0.000	0.333	0.667

Step 4 - Stack Weights

OSD-IP STACKED WEIGHTS

W1	W2	W3	W4
2.557	0.700	2.110	1.633

Step 5 - Calculate Weight Importance Factor

OSD-IP WIF

W1	W2	W3	W4
37%	10%	30%	23%

Step 6 - Calculate Scores

OSD-IP C-SCORES

Alternative	C-Score
A	0.490
B	0.679
C	0.780
D	0.550
E	0.536
F	0.787
G	0.614

Step 7 - Rank Alternatives

OSD-IP C-RANKS

Alternative	C-Rank
A	7
B	3
C	2
D	5
E	6
F	1
G	4

Capital City Site Selection [TONE00]

Seven Steps using OSD-DA

Step 1 - Collect & Orient Measures

Location	C1	C2	C3	C4
A	5	10	3	5
B	7	10	3	10
C	8	7	10	5
D	4	8	3	10
E	9	4	4	2
F	10	5	10	3
G	4	7	7	8

Step 2 - Level Scales

Alternative	C1	C2	C3	C4	Total
A	0.5	1	0.3	0.5	2.3
B	0.7	1	0.3	1	3.0
C	0.8	0.7	1	0.5	3.0
D	0.4	0.8	0.3	1	2.5
E	0.9	0.4	0.4	0.2	1.9
F	1	0.5	1	0.3	2.8
G	0.4	0.7	0.7	0.8	2.6

Step 3 - Choose Weights

OSD-DA RAW RESULTS

Alternative	W1	W2	W3	W4	Score	Average
A	0	1	0	0	5.100	0.729
B	0	0	0	1	4.300	0.614
C	0	0	1	0	4.000	0.571
D	0	0	0	1	4.300	0.614
E	1	0	0	0	5.222	0.746
F	0	0	1	0	4.000	0.571
G	0	0	0	1	5.375	0.768

OSD-DA NORM. RESULTS

Alternative	W1	W2	W3	W4
A	0	1	0	0
B	0	0	0	1
C	0	0	1	0
D	0	0	0	1
E	1	0	0	0
F	0	0	1	0
G	0	0	0	1

Step 4 - Stack Weights

OSD-DA STACKED WEIGHTS

W1	W2	W3	W4
1	1	2	3

Step 5 - Calculate Weight Importance Factor

OSD-DA WIF

W1	W2	W3	W4
14%	14%	29%	43%

Step 6 - Calculate Scores

OSD-DA C-SCORES

Alternative	C-Score
A	0.514
B	0.757
C	0.714
D	0.686
E	0.386
F	0.629
G	0.700

Step 7 - Rank Alternatives

OSD-DA C-RANKS

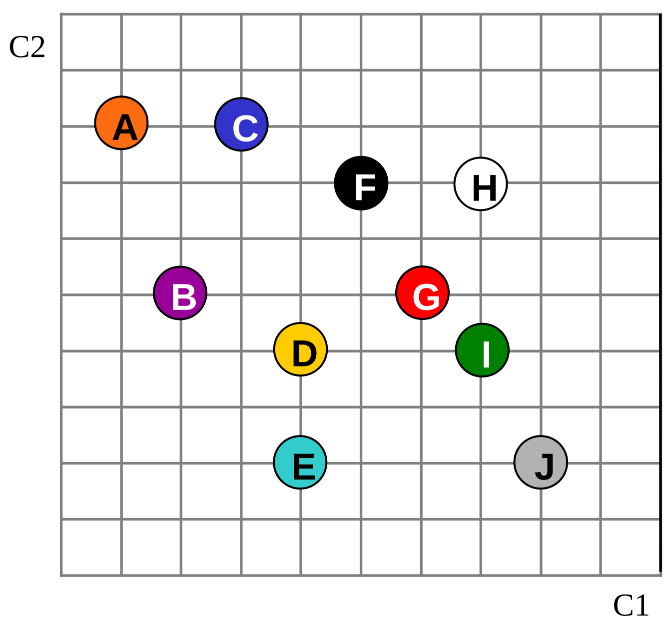
Alternative	C-Rank
A	6
B	1
C	2
D	4
E	7
F	5
G	3

APPENDIX D: 2D EXAMPLE LINDO PROGRAMS, OUTPUT & SUMMARY SHEETS

2D Example for Illustrative Purposes.

Raw Data:

Label	**Alternative**	**Criteria 1**	**Criteria 2**
A	Orange	1	8
B	Violet	2	5
C	Blue	3	8
D	Yellow	4	4
E	Cyan	4	2
F	Black	5	7
G	Red	6	5
H	White	7	7
I	Green	7	4
J	Grey	8	2



Lindo programs and output for OSD-CCR model

Orange – OSD-CCR Program

```
! Orange OSD-CCR Weights

max 0.1W1+0.8W2

st

0.1W1+0.8W2<=1
0.2W1+0.5W2<=1
0.3W1+0.8W2<=1
0.4W1+0.4W2<=1
0.4W1+0.2W2<=1
0.5W1+0.7W2<=1
0.6W1+0.5W2<=1
0.7W1+0.7W2<=1
0.7W1+0.4W2<=1
0.8W1+0.2W2<=1


W1>0
W2>0

End
```

Violet – OSD-CCR Program

```
! Violet OSD-CCR Weights

max 0.2W1+0.5W2

st

0.1W1+0.8W2<=1
0.2W1+0.5W2<=1
0.3W1+0.8W2<=1
0.4W1+0.4W2<=1
0.4W1+0.2W2<=1
0.5W1+0.7W2<=1
0.6W1+0.5W2<=1
0.7W1+0.7W2<=1
0.7W1+0.4W2<=1
0.8W1+0.2W2<=1


W1>0
W2>0

End
```

Blue – OSD-CCR Program

```
! Blue OSD-CCR Weights

max 0.3W1+0.8W2

st

0.1W1+0.8W2<=1
0.2W1+0.5W2<=1
0.3W1+0.8W2<=1
```

```
0.4W1+0.4W2<=1
0.4W1+0.2W2<=1
0.5W1+0.7W2<=1
0.6W1+0.5W2<=1
0.7W1+0.7W2<=1
0.7W1+0.4W2<=1
0.8W1+0.2W2<=1


W1>0
W2>0

End
```

Yellow – OSD-CCR Program

```
! Yellow OSD-CCR Weights

max 0.4W1+0.4W2

st

0.1W1+0.8W2<=1
0.2W1+0.5W2<=1
0.3W1+0.8W2<=1
0.4W1+0.4W2<=1
0.4W1+0.2W2<=1
0.5W1+0.7W2<=1
0.6W1+0.5W2<=1
0.7W1+0.7W2<=1
0.7W1+0.4W2<=1
0.8W1+0.2W2<=1


W1>0
W2>0

End
```

Cyan – OSD-CCR Program

```
! Cyan OSD-CCR Weights

max 0.4W1+0.2W2

st

0.1W1+0.8W2<=1
0.2W1+0.5W2<=1
0.3W1+0.8W2<=1
0.4W1+0.4W2<=1
0.4W1+0.2W2<=1
0.5W1+0.7W2<=1
0.6W1+0.5W2<=1
0.7W1+0.7W2<=1
0.7W1+0.4W2<=1
0.8W1+0.2W2<=1


W1>0
W2>0

End
```

Black – OSD-CCR Program

```
! Black OSD-CCR Weights

max 0.5W1+0.7W2

st

0.1W1+0.8W2<=1
0.2W1+0.5W2<=1
0.3W1+0.8W2<=1
0.4W1+0.4W2<=1
0.4W1+0.2W2<=1
0.5W1+0.7W2<=1
0.6W1+0.5W2<=1
0.7W1+0.7W2<=1
0.7W1+0.4W2<=1
0.8W1+0.2W2<=1


W1>0
W2>0

End
```

Red – OSD-CCR Program

```
! Red OSD-CCR Weights

max 0.6W1+0.5W2

st

0.1W1+0.8W2<=1
0.2W1+0.5W2<=1
0.3W1+0.8W2<=1
0.4W1+0.4W2<=1
0.4W1+0.2W2<=1
0.5W1+0.7W2<=1
0.6W1+0.5W2<=1
0.7W1+0.7W2<=1
0.7W1+0.4W2<=1
0.8W1+0.2W2<=1


W1>0
W2>0

End
```

White – OSD-CCR Program

```
! White OSD-CCR Weights

max 0.7W1+0.7W2

st

0.1W1+0.8W2<=1
0.2W1+0.5W2<=1
0.3W1+0.8W2<=1
0.4W1+0.4W2<=1
0.4W1+0.2W2<=1
0.5W1+0.7W2<=1
0.6W1+0.5W2<=1
```

```
0.7W1+0.7W2<=1
0.7W1+0.4W2<=1
0.8W1+0.2W2<=1


W1>0
W2>0

End
```

Green – OSD-CCR Program

```
! Green OSD-CCR Weights

max 0.7W1+0.4W2

st

0.1W1+0.8W2<=1
0.2W1+0.5W2<=1
0.3W1+0.8W2<=1
0.4W1+0.4W2<=1
0.4W1+0.2W2<=1
0.5W1+0.7W2<=1
0.6W1+0.5W2<=1
0.7W1+0.7W2<=1
0.7W1+0.4W2<=1
0.8W1+0.2W2<=1


W1>0
W2>0

End
```

Grey – OSD-CCR Program

```
! Grey OSD-CCR Weights

max 0.8W1+0.2W2

st

0.1W1+0.8W2<=1
0.2W1+0.5W2<=1
0.3W1+0.8W2<=1
0.4W1+0.4W2<=1
0.4W1+0.2W2<=1
0.5W1+0.7W2<=1
0.6W1+0.5W2<=1
0.7W1+0.7W2<=1
0.7W1+0.4W2<=1
0.8W1+0.2W2<=1


W1>0
W2>0

End
```

Orange – OSD-CCR Results

```
 LP OPTIMUM FOUND AT STEP      1
```

```
        OBJECTIVE FUNCTION VALUE

        1)      1.000000

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          0.200000
        W2         1.250000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          0.000000
        3)         0.375000          0.000000
        4)         0.000000          1.000000
        5)         0.500000          0.000000
        6)         0.750000          0.000000
        7)         0.125000          0.000000
        8)         0.375000          0.000000
        9)         0.125000          0.000000
       10)         0.500000          0.000000
       11)         0.750000          0.000000
       12)         0.000000          0.000000
       13)         1.250000          0.000000

 NO. ITERATIONS=       1
```

Violet – OSD-CCR Results

```
 LP OPTIMUM FOUND AT STEP      2

        OBJECTIVE FUNCTION VALUE

        1)     0.6285715

  VARIABLE        VALUE          REDUCED COST
        W1         0.285714          0.000000
        W2         1.142857          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.057143          0.000000
        3)         0.371429          0.000000
        4)         0.000000          0.600000
        5)         0.428571          0.000000
        6)         0.657143          0.000000
        7)         0.057143          0.000000
        8)         0.257143          0.000000
        9)         0.000000          0.028571
       10)         0.342857          0.000000
       11)         0.542857          0.000000
       12)         0.285714          0.000000
       13)         1.142857          0.000000

 NO. ITERATIONS=       2
```

Blue – OSD-CCR Results

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      1.000000
```

```
 VARIABLE        VALUE          REDUCED COST
       W1         0.000000          0.000000
       W2         1.250000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.000000
       3)         0.375000          0.000000
       4)         0.000000          1.000000
       5)         0.500000          0.000000
       6)         0.750000          0.000000
       7)         0.125000          0.000000
       8)         0.375000          0.000000
       9)         0.125000          0.000000
      10)         0.500000          0.000000
      11)         0.750000          0.000000
      12)         0.000000          0.000000
      13)         1.250000          0.000000

NO. ITERATIONS=       1
```

Yellow – OSD-CCR Results

```
LP OPTIMUM FOUND AT STEP      2

       OBJECTIVE FUNCTION VALUE

       1)     0.5714286

 VARIABLE        VALUE          REDUCED COST
       W1         1.190476          0.000000
       W2         0.238095          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.690476          0.000000
       3)         0.642857          0.000000
       4)         0.452381          0.000000
       5)         0.428571          0.000000
       6)         0.476190          0.000000
       7)         0.238095          0.000000
       8)         0.166667          0.000000
       9)         0.000000          0.571429
      10)         0.071429          0.000000
      11)         0.000000          0.000000
      12)         1.190476          0.000000
      13)         0.238095          0.000000

NO. ITERATIONS=       2
```

Cyan – OSD-CCR Results

```
LP OPTIMUM FOUND AT STEP      2

       OBJECTIVE FUNCTION VALUE

       1)     0.5238096

 VARIABLE        VALUE          REDUCED COST
       W1         1.190476          0.000000
       W2         0.238095          0.000000
```

```
      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.690476          0.000000
       3)         0.642857          0.000000
       4)         0.452381          0.000000
       5)         0.428571          0.000000
       6)         0.476190          0.000000
       7)         0.238095          0.000000
       8)         0.166667          0.000000
       9)         0.000000          0.190476
      10)         0.071429          0.000000
      11)         0.000000          0.333333
      12)         1.190476          0.000000
      13)         0.238095          0.000000

 NO. ITERATIONS=       2
```

Black – OSD-CCR Results

```
 LP OPTIMUM FOUND AT STEP      2

        OBJECTIVE FUNCTION VALUE

        1)      0.9428571

  VARIABLE        VALUE          REDUCED COST
        W1         0.285714          0.000000
        W2         1.142857          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.057143          0.000000
       3)         0.371429          0.000000
       4)         0.000000          0.400000
       5)         0.428571          0.000000
       6)         0.657143          0.000000
       7)         0.057143          0.000000
       8)         0.257143          0.000000
       9)         0.000000          0.542857
      10)         0.342857          0.000000
      11)         0.542857          0.000000
      12)         0.285714          0.000000
      13)         1.142857          0.000000

 NO. ITERATIONS=       2
```

Red – OSD-CCR Results

```
 LP OPTIMUM FOUND AT STEP      2

        OBJECTIVE FUNCTION VALUE

        1)      0.8333333

  VARIABLE        VALUE          REDUCED COST
        W1         1.190476          0.000000
        W2         0.238095          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.690476          0.000000
       3)         0.642857          0.000000
```

```
       4)         0.452381          0.000000
       5)         0.428571          0.000000
       6)         0.476190          0.000000
       7)         0.238095          0.000000
       8)         0.166667          0.000000
       9)         0.000000          0.666667
      10)         0.071429          0.000000
      11)         0.000000          0.166667
      12)         1.190476          0.000000
      13)         0.238095          0.000000

 NO. ITERATIONS=       2
```

White – OSD-CCR Results

```
 LP OPTIMUM FOUND AT STEP      2

        OBJECTIVE FUNCTION VALUE

        1)      1.000000

  VARIABLE        VALUE          REDUCED COST
        W1         1.190476          0.000000
        W2         0.238095          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.690476          0.000000
        3)         0.642857          0.000000
        4)         0.452381          0.000000
        5)         0.428571          0.000000
        6)         0.476190          0.000000
        7)         0.238095          0.000000
        8)         0.166667          0.000000
        9)         0.000000          1.000000
       10)         0.071429          0.000000
       11)         0.000000          0.000000
       12)         1.190476          0.000000
       13)         0.238095          0.000000

 NO. ITERATIONS=       2
```

Green – OSD-CCR Results

```
 LP OPTIMUM FOUND AT STEP      2

        OBJECTIVE FUNCTION VALUE

        1)     0.9285714

  VARIABLE        VALUE          REDUCED COST
        W1         1.190476          0.000000
        W2         0.238095          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.690476          0.000000
        3)         0.642857          0.000000
        4)         0.452381          0.000000
        5)         0.428571          0.000000
        6)         0.476190          0.000000
        7)         0.238095          0.000000
```

```
       8)          0.166667          0.000000
       9)          0.000000          0.428571
      10)          0.071429          0.000000
      11)          0.000000          0.500000
      12)          1.190476          0.000000
      13)          0.238095          0.000000

 NO. ITERATIONS=       2
```

Grey – OSD-CCR Results

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      1.000000

  VARIABLE        VALUE          REDUCED COST
        W1         1.250000          0.000000
        W2         0.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.875000          0.000000
        3)         0.750000          0.000000
        4)         0.625000          0.000000
        5)         0.500000          0.000000
        6)         0.500000          0.000000
        7)         0.375000          0.000000
        8)         0.250000          0.000000
        9)         0.125000          0.000000
       10)         0.125000          0.000000
       11)         0.000000          1.000000
       12)         1.250000          0.000000
       13)         0.000000          0.000000

 NO. ITERATIONS=       1
```

Lindo programs and output for OSD-IP model

Orange – OSD-IP Program

```
! Orange OSD-IP Weights

min Z1 + Z2 + Z3 + Z4 + Z5 + Z6 + Z7 + Z8 + Z9 + Z10

st

0.1W1+0.8W2-0.1W1-0.8W2-Z1<=0
0.2W1+0.5W2-0.1W1-0.8W2-Z2<=0
0.3W1+0.8W2-0.1W1-0.8W2-Z3<=0
0.4W1+0.4W2-0.1W1-0.8W2-Z4<=0
0.4W1+0.2W2-0.1W1-0.8W2-Z5<=0
0.5W1+0.7W2-0.1W1-0.8W2-Z6<=0
0.6W1+0.5W2-0.1W1-0.8W2-Z7<=0
0.7W1+0.7W2-0.1W1-0.8W2-Z8<=0
0.7W1+0.4W2-0.1W1-0.8W2-Z9<=0
0.8W1+0.2W2-0.1W1-0.8W2-Z10<=0

W1+W2 = 1
```

```
W1>0
W2>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
Integer Z7
Integer Z8
Integer Z9
Integer Z10
```

Violet – OSD-IP Program

```
! Violet OSD-IP Weights

min Z1 + Z2 + Z3 + Z4 + Z5 + Z6 + Z7 + Z8 + Z9 + Z10

st

0.1W1+0.8W2-0.2W1-0.5W2-Z1<=0
0.2W1+0.5W2-0.2W1-0.5W2-Z2<=0
0.3W1+0.8W2-0.2W1-0.5W2-Z3<=0
0.4W1+0.4W2-0.2W1-0.5W2-Z4<=0
0.4W1+0.2W2-0.2W1-0.5W2-Z5<=0
0.5W1+0.7W2-0.2W1-0.5W2-Z6<=0
0.6W1+0.5W2-0.2W1-0.5W2-Z7<=0
0.7W1+0.7W2-0.2W1-0.5W2-Z8<=0
0.7W1+0.4W2-0.2W1-0.5W2-Z9<=0
0.8W1+0.2W2-0.2W1-0.5W2-Z10<=0

W1+W2 = 1

W1>0
W2>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
Integer Z7
Integer Z8
Integer Z9
Integer Z10
```

Blue – OSD-IP Program

```
! Blue OSD-IP Weights

min Z1 + Z2 + Z3 + Z4 + Z5 + Z6 + Z7 + Z8 + Z9 + Z10

st

0.1W1+0.8W2-0.3W1-0.8W2-Z1<=0
0.2W1+0.5W2-0.3W1-0.8W2-Z2<=0
```

```
0.3W1+0.8W2-0.3W1-0.8W2-Z3<=0
0.4W1+0.4W2-0.3W1-0.8W2-Z4<=0
0.4W1+0.2W2-0.3W1-0.8W2-Z5<=0
0.5W1+0.7W2-0.3W1-0.8W2-Z6<=0
0.6W1+0.5W2-0.3W1-0.8W2-Z7<=0
0.7W1+0.7W2-0.3W1-0.8W2-Z8<=0
0.7W1+0.4W2-0.3W1-0.8W2-Z9<=0
0.8W1+0.2W2-0.3W1-0.8W2-Z10<=0

W1+W2 = 1

W1>0
W2>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
Integer Z7
Integer Z8
Integer Z9
Integer Z10
```

Yellow – OSD-IP Program

```
! Yellow OSD-IP Weights

min Z1 + Z2 + Z3 + Z4 + Z5 + Z6 + Z7 + Z8 + Z9 + Z10

st

0.1W1+0.8W2-0.4W1-0.4W2-Z1<=0
0.2W1+0.5W2-0.4W1-0.4W2-Z2<=0
0.3W1+0.8W2-0.4W1-0.4W2-Z3<=0
0.4W1+0.4W2-0.4W1-0.4W2-Z4<=0
0.4W1+0.2W2-0.4W1-0.4W2-Z5<=0
0.5W1+0.7W2-0.4W1-0.4W2-Z6<=0
0.6W1+0.5W2-0.4W1-0.4W2-Z7<=0
0.7W1+0.7W2-0.4W1-0.4W2-Z8<=0
0.7W1+0.4W2-0.4W1-0.4W2-Z9<=0
0.8W1+0.2W2-0.4W1-0.4W2-Z10<=0

W1+W2 = 1

W1>0
W2>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
Integer Z7
Integer Z8
Integer Z9
Integer Z10
```

Cyan – OSD-IP Program

```
! Cyan OSD-IP Weights

min Z1 + Z2 + Z3 + Z4 + Z5 + Z6 + Z7 + Z8 + Z9 + Z10

st

0.1W1+0.8W2-0.4W1-0.2W2-Z1<=0
0.2W1+0.5W2-0.4W1-0.2W2-Z2<=0
0.3W1+0.8W2-0.4W1-0.2W2-Z3<=0
0.4W1+0.4W2-0.4W1-0.2W2-Z4<=0
0.4W1+0.2W2-0.4W1-0.2W2-Z5<=0
0.5W1+0.7W2-0.4W1-0.2W2-Z6<=0
0.6W1+0.5W2-0.4W1-0.2W2-Z7<=0
0.7W1+0.7W2-0.4W1-0.2W2-Z8<=0
0.7W1+0.4W2-0.4W1-0.2W2-Z9<=0
0.8W1+0.2W2-0.4W1-0.2W2-Z10<=0

W1+W2 = 1

W1>0
W2>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
Integer Z7
Integer Z8
Integer Z9
Integer Z10
```

Black – OSD-IP Program

```
! Black OSD-IP Weights

min Z1 + Z2 + Z3 + Z4 + Z5 + Z6 + Z7 + Z8 + Z9 + Z10

st

0.1W1+0.8W2-0.5W1-0.7W2-Z1<=0
0.2W1+0.5W2-0.5W1-0.7W2-Z2<=0
0.3W1+0.8W2-0.5W1-0.7W2-Z3<=0
0.4W1+0.4W2-0.5W1-0.7W2-Z4<=0
0.4W1+0.2W2-0.5W1-0.7W2-Z5<=0
0.5W1+0.7W2-0.5W1-0.7W2-Z6<=0
0.6W1+0.5W2-0.5W1-0.7W2-Z7<=0
0.7W1+0.7W2-0.5W1-0.7W2-Z8<=0
0.7W1+0.4W2-0.5W1-0.7W2-Z9<=0
0.8W1+0.2W2-0.5W1-0.7W2-Z10<=0

W1+W2 = 1

W1>0
W2>0

end
```

```
Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
Integer Z7
Integer Z8
Integer Z9
Integer Z10
```

Red – OSD-IP Program

```
! Red OSD-IP Weights

min Z1 + Z2 + Z3 + Z4 + Z5 + Z6 + Z7 + Z8 + Z9 + Z10

st

0.1W1+0.8W2-0.6W1-0.5W2-Z1<=0
0.2W1+0.5W2-0.6W1-0.5W2-Z2<=0
0.3W1+0.8W2-0.6W1-0.5W2-Z3<=0
0.4W1+0.4W2-0.6W1-0.5W2-Z4<=0
0.4W1+0.2W2-0.6W1-0.5W2-Z5<=0
0.5W1+0.7W2-0.6W1-0.5W2-Z6<=0
0.6W1+0.5W2-0.6W1-0.5W2-Z7<=0
0.7W1+0.7W2-0.6W1-0.5W2-Z8<=0
0.7W1+0.4W2-0.6W1-0.5W2-Z9<=0
0.8W1+0.2W2-0.6W1-0.5W2-Z10<=0

W1+W2 = 1

W1>0
W2>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
Integer Z7
Integer Z8
Integer Z9
Integer Z10
```

White – OSD-IP Program

```
! White OSD-IP Weights

min Z1 + Z2 + Z3 + Z4 + Z5 + Z6 + Z7 + Z8 + Z9 + Z10

st

0.1W1+0.8W2-0.7W1-0.7W2-Z1<=0
0.2W1+0.5W2-0.7W1-0.7W2-Z2<=0
0.3W1+0.8W2-0.7W1-0.7W2-Z3<=0
0.4W1+0.4W2-0.7W1-0.7W2-Z4<=0
0.4W1+0.2W2-0.7W1-0.7W2-Z5<=0
0.5W1+0.7W2-0.7W1-0.7W2-Z6<=0
0.6W1+0.5W2-0.7W1-0.7W2-Z7<=0
0.7W1+0.7W2-0.7W1-0.7W2-Z8<=0
```

```
0.7W1+0.4W2-0.7W1-0.7W2-Z9<=0
0.8W1+0.2W2-0.7W1-0.7W2-Z10<=0

W1+W2 = 1

W1>0
W2>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
Integer Z7
Integer Z8
Integer Z9
Integer Z10
```

Green – OSD-IP Program

```
! Green OSD-IP Weights

min Z1 + Z2 + Z3 + Z4 + Z5 + Z6 + Z7 + Z8 + Z9 + Z10

st

0.1W1+0.8W2-0.7W1-0.4W2-Z1<=0
0.2W1+0.5W2-0.7W1-0.4W2-Z2<=0
0.3W1+0.8W2-0.7W1-0.4W2-Z3<=0
0.4W1+0.4W2-0.7W1-0.4W2-Z4<=0
0.4W1+0.2W2-0.7W1-0.4W2-Z5<=0
0.5W1+0.7W2-0.7W1-0.4W2-Z6<=0
0.6W1+0.5W2-0.7W1-0.4W2-Z7<=0
0.7W1+0.7W2-0.7W1-0.4W2-Z8<=0
0.7W1+0.4W2-0.7W1-0.4W2-Z9<=0
0.8W1+0.2W2-0.7W1-0.4W2-Z10<=0

W1+W2 = 1

W1>0
W2>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
Integer Z7
Integer Z8
Integer Z9
Integer Z10
```

Grey – OSD-IP Program

```
! Grey OSD-IP Weights

min Z1 + Z2 + Z3 + Z4 + Z5 + Z6 + Z7 + Z8 + Z9 + Z10
```

```
st

0.1W1+0.8W2-0.8W1-0.2W2-Z1<=0
0.2W1+0.5W2-0.8W1-0.2W2-Z2<=0
0.3W1+0.8W2-0.8W1-0.2W2-Z3<=0
0.4W1+0.4W2-0.8W1-0.2W2-Z4<=0
0.4W1+0.2W2-0.8W1-0.2W2-Z5<=0
0.5W1+0.7W2-0.8W1-0.2W2-Z6<=0
0.6W1+0.5W2-0.8W1-0.2W2-Z7<=0
0.7W1+0.7W2-0.8W1-0.2W2-Z8<=0
0.7W1+0.4W2-0.8W1-0.2W2-Z9<=0
0.8W1+0.2W2-0.8W1-0.2W2-Z10<=0

W1+W2 = 1

W1>0
W2>0

end

Integer Z1
Integer Z2
Integer Z3
Integer Z4
Integer Z5
Integer Z6
Integer Z7
Integer Z8
Integer Z9
Integer Z10
```

Orange – OSD-IP Results

```
 LP OPTIMUM FOUND AT STEP      0
 OBJECTIVE VALUE =  0.000000000E+00

 FIX ALL VARS.(   10)  WITH RC >   1.00000

 NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       0
 BOUND ON OPTIMUM: 0.0000000E+00
 ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       0

 LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)     0.0000000E+00

  VARIABLE        VALUE          REDUCED COST
        Z1         0.000000          1.000000
        Z2         0.000000          1.000000
        Z3         0.000000          1.000000
        Z4         0.000000          1.000000
        Z5         0.000000          1.000000
        Z6         0.000000          1.000000
        Z7         0.000000          1.000000
        Z8         0.000000          1.000000
        Z9         0.000000          1.000000
       Z10         0.000000          1.000000
```

```
       W1         0.000000          0.000000
       W2         1.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.000000
       3)         0.300000          0.000000
       4)         0.000000          0.000000
       5)         0.400000          0.000000
       6)         0.600000          0.000000
       7)         0.100000          0.000000
       8)         0.300000          0.000000
       9)         0.100000          0.000000
      10)         0.400000          0.000000
      11)         0.600000          0.000000
      12)         0.000000          0.000000
      13)         0.000000          0.000000
      14)         1.000000          0.000000

 NO. ITERATIONS=       0
 BRANCHES=    0 DETERM.=  1.000E    0
```

Violet – OSD-IP Results

```
 LP OPTIMUM FOUND AT STEP      5
 OBJECTIVE VALUE =   1.00000000

 FIX ALL VARS.(    6)  WITH RC >  0.500000
 SET       Z1 TO >=     1 AT    1, BND=  -1.700     TWIN=-0.1000E+31     12
 SET       Z3 TO >=     1 AT    2, BND=  -2.400     TWIN=-0.1000E+31     14
 SET       Z6 TO >=     1 AT    3, BND=  -3.200     TWIN=-0.1000E+31     15
 SET       Z8 TO >=     1 AT    4, BND=  -4.000     TWIN=-0.1000E+31     16

 NEW INTEGER SOLUTION OF    4.00000000     AT BRANCH      4 PIVOT      16
 BOUND ON OPTIMUM:  1.500000
 DELETE       Z8 AT LEVEL     4
 DELETE       Z6 AT LEVEL     3
 DELETE       Z3 AT LEVEL     2
 DELETE       Z1 AT LEVEL     1
 RELEASE FIXED VARIABLES
 SET       Z8 TO >=     1 AT    1, BND=  -2.467     TWIN=-0.1000E+31     47
 SET      Z10 TO <=     0 AT    2, BND=  -2.633     TWIN= -2.783         54
 SET       Z3 TO >=     1 AT    3, BND=  -3.400     TWIN=-0.1000E+31     55
 DELETE       Z3 AT LEVEL     3
 FLIP      Z10 TO >=         1 AT    2 WITH BND=   -2.7833333
 SET       Z2 TO <=     0 AT    3, BND=  -2.783     TWIN=-0.1000E+31     55
 SET       Z4 TO <=     0 AT    4, BND=  -2.783     TWIN=-0.1000E+31     55
 SET       Z5 TO <=     0 AT    5, BND=  -2.783     TWIN=-0.1000E+31     55
 SET       Z9 TO <=     0 AT    6, BND=  -2.783     TWIN=-0.1000E+31     55
 SET       Z3 TO >=     1 AT    7, BND=  -3.500     TWIN=-0.1000E+31     58
 DELETE       Z3 AT LEVEL     7
 DELETE       Z9 AT LEVEL     6
 DELETE       Z5 AT LEVEL     5
 DELETE       Z4 AT LEVEL     4
 DELETE       Z2 AT LEVEL     3
 DELETE      Z10 AT LEVEL     2
 DELETE       Z8 AT LEVEL     1
 ENUMERATION COMPLETE. BRANCHES=     8 PIVOTS=      58

 LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...
```

```
       OBJECTIVE FUNCTION VALUE

       1)      4.000000

 VARIABLE        VALUE          REDUCED COST
       Z1         1.000000          1.000000
       Z2         0.000000          1.000000
       Z3         1.000000          1.000000
       Z4         0.000000          1.000000
       Z5         0.000000          1.000000
       Z6         1.000000          1.000000
       Z7         0.000000          1.000000
       Z8         1.000000          1.000000
       Z9         0.000000          1.000000
      Z10         0.000000          1.000000
       W1         0.000000          0.000000
       W2         1.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.700000          0.000000
       3)         0.000000          0.000000
       4)         0.700000          0.000000
       5)         0.100000          0.000000
       6)         0.300000          0.000000
       7)         0.800000          0.000000
       8)         0.000000          0.000000
       9)         0.800000          0.000000
      10)         0.100000          0.000000
      11)         0.300000          0.000000
      12)         0.000000          0.000000
      13)         0.000000          0.000000
      14)         1.000000          0.000000

NO. ITERATIONS=      59
BRANCHES=    8 DETERM.=  1.000E    0
```

Blue – OSD-IP Results

```
LP OPTIMUM FOUND AT STEP      0
OBJECTIVE VALUE =  0.000000000E+00

FIX ALL VARS.(   10)  WITH RC >   1.00000

NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT      0
BOUND ON OPTIMUM: 0.0000000E+00
ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=      0

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)     0.0000000E+00

 VARIABLE        VALUE          REDUCED COST
       Z1         0.000000          1.000000
       Z2         0.000000          1.000000
       Z3         0.000000          1.000000
       Z4         0.000000          1.000000
       Z5         0.000000          1.000000
       Z6         0.000000          1.000000
```

```
       Z7          0.000000          1.000000
       Z8          0.000000          1.000000
       Z9          0.000000          1.000000
      Z10          0.000000          1.000000
       W1          0.000000          0.000000
       W2          1.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)          0.000000          0.000000
       3)          0.300000          0.000000
       4)          0.000000          0.000000
       5)          0.400000          0.000000
       6)          0.600000          0.000000
       7)          0.100000          0.000000
       8)          0.300000          0.000000
       9)          0.100000          0.000000
      10)          0.400000          0.000000
      11)          0.600000          0.000000
      12)          0.000000          0.000000
      13)          0.000000          0.000000
      14)          1.000000          0.000000

NO. ITERATIONS=       0
BRANCHES=    0 DETERM.=  1.000E    0
```

Yellow – OSD-IP Results

```
LP OPTIMUM FOUND AT STEP      7
OBJECTIVE VALUE =   1.07142854

FIX ALL VARS.(    4)  WITH RC >  0.571429
SET       Z8 TO >=     1 AT    1, BND=  -1.771     TWIN=-0.1000E+31     10
SET       Z6 TO >=     1 AT    2, BND=  -2.586     TWIN=-0.1000E+31     13
SET       Z9 TO >=     1 AT    3, BND=  -3.414     TWIN=-0.1000E+31     14
SET       Z7 TO >=     1 AT    4, BND=  -4.257     TWIN=-0.1000E+31     15
SET      Z10 TO >=     1 AT    5, BND=  -5.000     TWIN=-0.1000E+31     18

NEW INTEGER SOLUTION OF    5.00000000     AT BRANCH      5 PIVOT      18
BOUND ON OPTIMUM:  1.642857
DELETE      Z10 AT LEVEL     5
DELETE       Z7 AT LEVEL     4
DELETE       Z9 AT LEVEL     3
DELETE       Z6 AT LEVEL     2
DELETE       Z8 AT LEVEL     1
RELEASE FIXED VARIABLES
SET       Z8 TO >=     1 AT    1, BND=  -2.700     TWIN=-0.1000E+31     28
SET       Z6 TO >=     1 AT    2, BND=  -3.467     TWIN=-0.1000E+31     31
SET      Z10 TO <=     0 AT    3, BND=  -3.467     TWIN=-0.1000E+31     31
SET       Z3 TO >=     1 AT    4, BND=  -4.100     TWIN=-0.1000E+31     35
DELETE       Z3 AT LEVEL     4
DELETE      Z10 AT LEVEL     3
DELETE       Z6 AT LEVEL     2
DELETE       Z8 AT LEVEL     1
ENUMERATION COMPLETE. BRANCHES=     8 PIVOTS=      35

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)      5.000000
```

```
 VARIABLE        VALUE          REDUCED COST
       Z1         0.000000          1.000000
       Z2         0.000000          1.000000
       Z3         0.000000          1.000000
       Z4         0.000000          1.000000
       Z5         0.000000          1.000000
       Z6         1.000000          1.000000
       Z7         1.000000          1.000000
       Z8         1.000000          1.000000
       Z9         1.000000          1.000000
      Z10         1.000000          1.000000
       W1         0.800000          0.000000
       W2         0.200000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.160000          0.000000
       3)         0.140000          0.000000
       4)         0.000000          0.000000
       5)         0.000000          0.000000
       6)         0.040000          0.000000
       7)         0.860000          0.000000
       8)         0.820000          0.000000
       9)         0.700000          0.000000
      10)         0.760000          0.000000
      11)         0.720000          0.000000
      12)         0.000000          0.000000
      13)         0.800000          0.000000
      14)         0.200000          0.000000

NO. ITERATIONS=      36
BRANCHES=    8 DETERM.=  1.000E    0
```

Cyan – OSD-IP Results

```
LP OPTIMUM FOUND AT STEP      5
OBJECTIVE VALUE =   1.29999995

FIX ALL VARS.(    5)  WITH RC >   1.00000
SET      Z10 TO >=     1 AT    1, BND=  -1.900     TWIN=-0.1000E+31     11
SET       Z8 TO >=     1 AT    2, BND=  -2.600     TWIN=-0.1000E+31     12
SET       Z9 TO >=     1 AT    3, BND=  -3.300     TWIN=-0.1000E+31     14
SET       Z7 TO >=     1 AT    4, BND=  -4.100     TWIN=-0.1000E+31     15
SET       Z6 TO >=     1 AT    5, BND=  -5.000     TWIN=-0.1000E+31     16

NEW INTEGER SOLUTION OF    5.00000000     AT BRANCH      5 PIVOT      16
BOUND ON OPTIMUM:  2.300000
DELETE       Z6 AT LEVEL     5
DELETE       Z7 AT LEVEL     4
DELETE       Z9 AT LEVEL     3
DELETE       Z8 AT LEVEL     2
DELETE      Z10 AT LEVEL     1
RELEASE FIXED VARIABLES
SET      Z10 TO <=     0 AT    1, BND=  -3.200     TWIN= -2.900         42
SET       Z5 TO <=     0 AT    2, BND=  -3.200     TWIN=-0.1000E+31     42
SET       Z3 TO >=     1 AT    3, BND=  -3.600     TWIN=-0.1000E+31     43
SET       Z6 TO >=     1 AT    4, BND=  -4.100     TWIN=-0.1000E+31     44
DELETE       Z6 AT LEVEL     4
DELETE       Z3 AT LEVEL     3
DELETE       Z5 AT LEVEL     2
FLIP      Z10 TO >=         1 AT    1 WITH BND=   -2.9000001
```

```
SET       Z8 TO >=     1 AT    2, BND=  -3.600     TWIN=-0.1000E+31     45
SET       Z9 TO >=     1 AT    3, BND=  -4.300     TWIN=-0.1000E+31     51
DELETE       Z9 AT LEVEL     3
DELETE       Z8 AT LEVEL     2
DELETE      Z10 AT LEVEL     1
ENUMERATION COMPLETE. BRANCHES=    10 PIVOTS=      51

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)      5.000000

 VARIABLE        VALUE          REDUCED COST
       Z1         0.000000          1.000000
       Z2         0.000000          1.000000
       Z3         0.000000          1.000000
       Z4         0.000000          1.000000
       Z5         0.000000          1.000000
       Z6         1.000000          1.000000
       Z7         1.000000          1.000000
       Z8         1.000000          1.000000
       Z9         1.000000          1.000000
      Z10         1.000000          1.000000
       W1         1.000000          0.000000
       W2         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.300000          0.000000
       3)         0.200000          0.000000
       4)         0.100000          0.000000
       5)         0.000000          0.000000
       6)         0.000000          0.000000
       7)         0.900000          0.000000
       8)         0.800000          0.000000
       9)         0.700000          0.000000
      10)         0.700000          0.000000
      11)         0.600000          0.000000
      12)         0.000000          0.000000
      13)         1.000000          0.000000
      14)         0.000000          0.000000

NO. ITERATIONS=      54
BRANCHES=   10 DETERM.=  1.000E    0
```

Black – OSD-IP Results

```
LP OPTIMUM FOUND AT STEP      6
OBJECTIVE VALUE =  0.666666776E-01

FIX ALL VARS.(    9)  WITH RC >  0.333333
SET       Z8 TO >=     1 AT    1, BND=  -1.000     TWIN=-0.1000E+31     13

NEW INTEGER SOLUTION OF    1.00000000     AT BRANCH      1 PIVOT      13
BOUND ON OPTIMUM: 0.4000000
DELETE       Z8 AT LEVEL     1
ENUMERATION COMPLETE. BRANCHES=     1 PIVOTS=      13

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...
```

```
       OBJECTIVE FUNCTION VALUE

       1)      1.000000

 VARIABLE        VALUE          REDUCED COST
       Z1         0.000000          1.000000
       Z2         0.000000          1.000000
       Z3         0.000000          1.000000
       Z4         0.000000          1.000000
       Z5         0.000000          1.000000
       Z6         0.000000          1.000000
       Z7         0.000000          1.000000
       Z8         1.000000          1.000000
       Z9         0.000000          1.000000
      Z10         0.000000          1.000000
       W1         0.600000          0.000000
       W2         0.400000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.200000          0.000000
       3)         0.260000          0.000000
       4)         0.080000          0.000000
       5)         0.180000          0.000000
       6)         0.260000          0.000000
       7)         0.000000          0.000000
       8)         0.020000          0.000000
       9)         0.880000          0.000000
      10)         0.000000          0.000000
      11)         0.020000          0.000000
      12)         0.000000          0.000000
      13)         0.600000          0.000000
      14)         0.400000          0.000000

NO. ITERATIONS=      14
BRANCHES=    1 DETERM.=  1.000E    0
```

Red – OSD-IP Results

```
LP OPTIMUM FOUND AT STEP      7
OBJECTIVE VALUE =  0.179999948

FIX ALL VARS.(    7)  WITH RC >  0.600000
SET       Z8 TO >=     1 AT    1, BND=  -1.040     TWIN=-0.1000E+31     12
SET       Z6 TO >=     1 AT    2, BND=  -2.000     TWIN=-0.1000E+31     15

NEW INTEGER SOLUTION OF    2.00000000     AT BRANCH      2 PIVOT      15
BOUND ON OPTIMUM: 0.7799997
DELETE       Z6 AT LEVEL     2
DELETE       Z8 AT LEVEL     1
RELEASE FIXED VARIABLES
ENUMERATION COMPLETE. BRANCHES=     2 PIVOTS=      20

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)      2.000000

 VARIABLE        VALUE          REDUCED COST
```

```
      Z1         0.000000          1.000000
      Z2         0.000000          1.000000
      Z3         0.000000          1.000000
      Z4         0.000000          1.000000
      Z5         0.000000          1.000000
      Z6         1.000000          1.000000
      Z7         0.000000          1.000000
      Z8         1.000000          1.000000
      Z9         0.000000          1.000000
     Z10         0.000000          1.000000
      W1         0.500000          0.000000
      W2         0.500000          0.000000


     ROW   SLACK OR SURPLUS     DUAL PRICES
      2)         0.100000          0.000000
      3)         0.200000          0.000000
      4)         0.000000          0.000000
      5)         0.150000          0.000000
      6)         0.250000          0.000000
      7)         0.950000          0.000000
      8)         0.000000          0.000000
      9)         0.850000          0.000000
     10)         0.000000          0.000000
     11)         0.050000          0.000000
     12)         0.000000          0.000000
     13)         0.500000          0.000000
     14)         0.500000          0.000000

NO. ITERATIONS=      21
BRANCHES=    2 DETERM.=  1.000E    0
```

White – OSD-IP Results

```
LP OPTIMUM FOUND AT STEP      1
OBJECTIVE VALUE =  0.000000000E+00

FIX ALL VARS.(   10)  WITH RC >   1.00000

NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       1
BOUND ON OPTIMUM: 0.0000000E+00
ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       1

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)     0.0000000E+00

 VARIABLE        VALUE          REDUCED COST
      Z1         0.000000          1.000000
      Z2         0.000000          1.000000
      Z3         0.000000          1.000000
      Z4         0.000000          1.000000
      Z5         0.000000          1.000000
      Z6         0.000000          1.000000
      Z7         0.000000          1.000000
      Z8         0.000000          1.000000
      Z9         0.000000          1.000000
     Z10         0.000000          1.000000
      W1         0.833333          0.000000
```

```
       W2         0.166667          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.483333          0.000000
       3)         0.450000          0.000000
       4)         0.316667          0.000000
       5)         0.300000          0.000000
       6)         0.333333          0.000000
       7)         0.166667          0.000000
       8)         0.116667          0.000000
       9)         0.000000          0.000000
      10)         0.050000          0.000000
      11)         0.000000          0.000000
      12)         0.000000          0.000000
      13)         0.833333          0.000000
      14)         0.166667          0.000000

NO. ITERATIONS=       1
BRANCHES=    0 DETERM.=  1.000E    0
```

Green – OSD-IP Results

```
LP OPTIMUM FOUND AT STEP      3
OBJECTIVE VALUE =  0.100000024

FIX ALL VARS.(    9)  WITH RC >  0.000000E+00
SET      Z10 TO >=     1 AT    1, BND=  -1.000     TWIN=-0.1000E+31      8

NEW INTEGER SOLUTION OF    1.00000000     AT BRANCH      1 PIVOT       8
BOUND ON OPTIMUM: 0.1000000
DELETE      Z10 AT LEVEL     1
ENUMERATION COMPLETE. BRANCHES=     1 PIVOTS=       8

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)      1.000000

 VARIABLE        VALUE          REDUCED COST
       Z1         0.000000          1.000000
       Z2         0.000000          1.000000
       Z3         0.000000          1.000000
       Z4         0.000000          1.000000
       Z5         0.000000          1.000000
       Z6         0.000000          1.000000
       Z7         0.000000          1.000000
       Z8         0.000000          1.000000
       Z9         0.000000          1.000000
      Z10         1.000000          1.000000
       W1         1.000000          0.000000
       W2         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.600000          0.000000
       3)         0.500000          0.000000
       4)         0.400000          0.000000
       5)         0.300000          0.000000
       6)         0.300000          0.000000
```

```
       7)          0.200000          0.000000
       8)          0.100000          0.000000
       9)          0.000000          0.000000
      10)          0.000000          0.000000
      11)          0.900000          0.000000
      12)          0.000000          0.000000
      13)          1.000000          0.000000
      14)          0.000000          0.000000

 NO. ITERATIONS=       8
 BRANCHES=    1 DETERM.=  1.000E    0
```

Grey – OSD-IP Results

```
 LP OPTIMUM FOUND AT STEP      1
 OBJECTIVE VALUE =  0.000000000E+00

 FIX ALL VARS.(   10)  WITH RC >   1.00000

 NEW INTEGER SOLUTION OF   0.000000000E+00 AT BRANCH      0 PIVOT       1
 BOUND ON OPTIMUM: 0.0000000E+00
 ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=       1

 LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)     0.0000000E+00

  VARIABLE        VALUE          REDUCED COST
        Z1         0.000000          1.000000
        Z2         0.000000          1.000000
        Z3         0.000000          1.000000
        Z4         0.000000          1.000000
        Z5         0.000000          1.000000
        Z6         0.000000          1.000000
        Z7         0.000000          1.000000
        Z8         0.000000          1.000000
        Z9         0.000000          1.000000
       Z10         0.000000          1.000000
        W1         0.833333          0.000000
        W2         0.166667          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.483333          0.000000
        3)         0.450000          0.000000
        4)         0.316667          0.000000
        5)         0.300000          0.000000
        6)         0.333333          0.000000
        7)         0.166667          0.000000
        8)         0.116667          0.000000
        9)         0.000000          0.000000
       10)         0.050000          0.000000
       11)         0.000000          0.000000
       12)         0.000000          0.000000
       13)         0.833333          0.000000
       14)         0.166667          0.000000

 NO. ITERATIONS=       1
 BRANCHES=    0 DETERM.=  1.000E    0
```

Lindo programs and output for OSD-DA model

Orange – OSD-DA Program

```
! Orange OSD-DA Weights

min
0.1W1+0.8W2+0.2W1+0.5W2+0.3W1+0.8W2+0.4W1+0.4W2+0.4W1+0.2W2+0.5W1+0.7W2+0.6
W1+0.5W2+0.7W1+0.7W2+0.7W1+0.4W2+0.8W1+0.2W2

st

0.1W1+0.8W2=1

W1>0
W2>0

End
```

Violet – OSD-DA Program

```
! Violet OSD-DA Weights

min
0.1W1+0.8W2+0.2W1+0.5W2+0.3W1+0.8W2+0.4W1+0.4W2+0.4W1+0.2W2+0.5W1+0.7W2+0.6
W1+0.5W2+0.7W1+0.7W2+0.7W1+0.4W2+0.8W1+0.2W2

st

0.2W1+0.5W2=1

W1>0
W2>0

End
```

Blue – OSD-DA Program

```
! Blue OSD-DA Weights

min
0.1W1+0.8W2+0.2W1+0.5W2+0.3W1+0.8W2+0.4W1+0.4W2+0.4W1+0.2W2+0.5W1+0.7W2+0.6
W1+0.5W2+0.7W1+0.7W2+0.7W1+0.4W2+0.8W1+0.2W2

st

0.3W1+0.8W2=1

W1>0
W2>0

End
```

Yellow – OSD-DA Program

```
! Yellow OSD-DA Weights

min
0.1W1+0.8W2+0.2W1+0.5W2+0.3W1+0.8W2+0.4W1+0.4W2+0.4W1+0.2W2+0.5W1+0.7W2+0.6
W1+0.5W2+0.7W1+0.7W2+0.7W1+0.4W2+0.8W1+0.2W2

st
```

```
0.4W1+0.4W2=1

W1>0
W2>0

End
```

Cyan – OSD-DA Program

```
! Cyan OSD-DA Weights

min
0.1W1+0.8W2+0.2W1+0.5W2+0.3W1+0.8W2+0.4W1+0.4W2+0.4W1+0.2W2+0.5W1+0.7W2+0.6
W1+0.5W2+0.7W1+0.7W2+0.7W1+0.4W2+0.8W1+0.2W2

st

0.4W1+0.2W2=1

W1>0
W2>0

End
```

Black – OSD-DA Program

```
! Black OSD-DA Weights

min
0.1W1+0.8W2+0.2W1+0.5W2+0.3W1+0.8W2+0.4W1+0.4W2+0.4W1+0.2W2+0.5W1+0.7W2+0.6
W1+0.5W2+0.7W1+0.7W2+0.7W1+0.4W2+0.8W1+0.2W2

st

0.5W1+0.7W2=1

W1>0
W2>0

End
```

Red – OSD-DA Program

```
! Red OSD-DA Weights

min
0.1W1+0.8W2+0.2W1+0.5W2+0.3W1+0.8W2+0.4W1+0.4W2+0.4W1+0.2W2+0.5W1+0.7W2+0.6
W1+0.5W2+0.7W1+0.7W2+0.7W1+0.4W2+0.8W1+0.2W2

st

0.6W1+0.5W2=1

W1>0
W2>0

End
```

White – OSD-DA Program

```
! White OSD-DA Weights

min
0.1W1+0.8W2+0.2W1+0.5W2+0.3W1+0.8W2+0.4W1+0.4W2+0.4W1+0.2W2+0.5W1+0.7W2+0.6
W1+0.5W2+0.7W1+0.7W2+0.7W1+0.4W2+0.8W1+0.2W2
```

```
st

0.7W1+0.7W2=1

W1>0
W2>0

End
```

Green – OSD-DA Program

```
! Green OSD-DA Weights

min
0.1W1+0.8W2+0.2W1+0.5W2+0.3W1+0.8W2+0.4W1+0.4W2+0.4W1+0.2W2+0.5W1+0.7W2+0.6
W1+0.5W2+0.7W1+0.7W2+0.7W1+0.4W2+0.8W1+0.2W2

st

0.7W1+0.4W2=1

W1>0
W2>0

End
```

Grey – OSD-DA Program

```
! Grey OSD-DA Weights

min
0.1W1+0.8W2+0.2W1+0.5W2+0.3W1+0.8W2+0.4W1+0.4W2+0.4W1+0.2W2+0.5W1+0.7W2+0.6
W1+0.5W2+0.7W1+0.7W2+0.7W1+0.4W2+0.8W1+0.2W2

st

0.8W1+0.2W2=1

W1>0
W2>0

end
```

Orange – OSD-DA Results

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      6.500000

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          4.050000
        W2         1.250000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -6.500000
        3)         0.000000          0.000000
        4)         1.250000          0.000000

 NO. ITERATIONS=       1
```

Violet – OSD-DA Results

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      10.40000

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          2.620000
        W2         2.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000        -10.400000
        3)         0.000000          0.000000
        4)         2.000000          0.000000

 NO. ITERATIONS=       1
```

Blue – OSD-DA Results

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      6.500000

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          2.750000
        W2         1.250000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -6.500000
        3)         0.000000          0.000000
        4)         1.250000          0.000000

 NO. ITERATIONS=       1
```

Yellow – OSD-DA Results

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      11.75000

  VARIABLE        VALUE          REDUCED COST
        W1         2.500000          0.000000
        W2         0.000000          0.500000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000        -11.750000
        3)         2.500000          0.000000
        4)         0.000000          0.000000

 NO. ITERATIONS=       1
```

Cyan – OSD-DA Results

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      11.75000

  VARIABLE        VALUE          REDUCED COST
        W1         2.500000          0.000000
        W2         0.000000          2.850000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000        -11.750000
        3)         2.500000          0.000000
        4)         0.000000          0.000000

 NO. ITERATIONS=       1
```

Black – OSD-DA Results

```
 LP OPTIMUM FOUND AT STEP      0

        OBJECTIVE FUNCTION VALUE

        1)      7.428571

  VARIABLE        VALUE          REDUCED COST
        W1         0.000000          0.985715
        W2         1.428571          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -7.428571
        3)         0.000000          0.000000
        4)         1.428571          0.000000

 NO. ITERATIONS=       0
```

Red – OSD-DA Results

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      7.833333

  VARIABLE        VALUE          REDUCED COST
        W1         1.666667          0.000000
        W2         0.000000          1.283333


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000         -7.833333
        3)         1.666667          0.000000
        4)         0.000000          0.000000

 NO. ITERATIONS=       1
```

White – OSD-DA Results

```
 LP OPTIMUM FOUND AT STEP      1
```

```
       OBJECTIVE FUNCTION VALUE

       1)      6.714286

 VARIABLE        VALUE          REDUCED COST
       W1         1.428571          0.000000
       W2         0.000000          0.500000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000         -6.714286
       3)         1.428571          0.000000
       4)         0.000000          0.000000

 NO. ITERATIONS=       1
```

Green – OSD-DA Results

```
 LP OPTIMUM FOUND AT STEP      1

       OBJECTIVE FUNCTION VALUE

       1)      6.714286

 VARIABLE        VALUE          REDUCED COST
       W1         1.428571          0.000000
       W2         0.000000          2.514286


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000         -6.714286
       3)         1.428571          0.000000
       4)         0.000000          0.000000

 NO. ITERATIONS=       1
```

Grey – OSD-DA Results

```
 LP OPTIMUM FOUND AT STEP      1

       OBJECTIVE FUNCTION VALUE

       1)      5.875000

 VARIABLE        VALUE          REDUCED COST
       W1         1.250000          0.000000
       W2         0.000000          4.025000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000         -5.875000
       3)         1.250000          0.000000
       4)         0.000000          0.000000

 NO. ITERATIONS=       1
```

2D Example

Seven Steps using OSD-CCR

Step 1 - Collect Measures

Alternative	C1	C2
Orange	1	8
Violet	2	5
Blue	3	8
Yellow	4	4
Cyan	4	2
Black	5	7
Red	6	5
White	7	7
Green	7	4
Grey	8	2

Step 2 - Level Scales

Alternative	C1	C2
Orange	0.1	0.8
Violet	0.2	0.5
Blue	0.3	0.8
Yellow	0.4	0.4
Cyan	0.4	0.2
Black	0.5	0.7
Red	0.6	0.5
White	0.7	0.7
Green	0.7	0.4
Grey	0.8	0.2

Step 3a - Choose Weights

OSD-CCR Raw Weights

Alternative	W1	W2	Score
Orange	0.000	1.250	1.000
Violet	0.286	1.143	0.629
Blue	0.000	1.250	1.000
Yellow	1.190	0.238	0.571
Cyan	1.190	0.238	0.524
Black	0.286	1.143	0.943
Red	1.190	0.238	0.833
White	1.190	0.238	1.000
Green	1.190	0.238	0.929
Grey	1.250	0.000	1.000

Step 3b - Choose Weights

OSD-CCR Normalized Weights

Alternative	W1	W2
Orange	0.000	1.000
Violet	0.200	0.800
Blue	0.000	1.000
Yellow	0.833	0.167
Cyan	0.833	0.167
Black	0.200	0.800
Red	0.833	0.167
White	0.833	0.167
Green	0.833	0.167
Grey	1.000	0.000

Step 4 - Stack Weights

OSD-IP Stacked Weights

5.567	4.433

Step 5 - Calculate WIF

OSD-IP Normalized Weights

56%	44%

Step 6 - Calculate Scores

Alternative	C-Score
Orange	0.410
Violet	0.333
Blue	0.522
Yellow	0.400
Cyan	0.311
Black	0.589
Red	0.556
White	0.700
Green	0.567
Grey	0.534

Step 7 - Rank Alternatives

Alternative	C-Rank
Orange	7
Violet	9
Blue	6
Yellow	8
Cyan	10
Black	2
Red	4
White	1
Green	3
Grey	5

2D Example

Seven Steps using OSD-IP

Step 1 - Collect Measures

Alternative	C1	C2
Orange	1	8
Violet	2	5
Blue	3	8
Yellow	4	4
Cyan	4	2
Black	5	7
Red	6	5
White	7	7
Green	7	4
Grey	8	2

Step 2 - Level Scales

Alternative	C1	C2
Orange	0.1	0.8
Violet	0.2	0.5
Blue	0.3	0.8
Yellow	0.4	0.4
Cyan	0.4	0.2
Black	0.5	0.7
Red	0.6	0.5
White	0.7	0.7
Green	0.7	0.4
Grey	0.8	0.2

Step 3 - Choose Weights

Alternative	W1	W2	No. Above	Max Rank
Orange	0.000	1.000	0	1
Violet	0.000	1.000	4	5
Blue	0.000	1.000	0	1
Yellow	0.800	0.200	5	6
Cyan	1.000	0.000	5	6
Black	0.600	0.400	1	2
Red	0.500	0.500	2	3
White	0.833	0.167	0	1
Green	1.000	0.000	1	2
Grey	0.833	0.167	0	1

Step 4 - Stack Weights

OSD-IP Stacked Weights		
	5.567	4.433

Step 5 - Calculate WIF

OSD-IP Normalized Weights		
	56%	44%

Step 6 - Calculate Scores

Alternative	C-Score
Orange	0.410
Violet	0.333
Blue	0.522
Yellow	0.400
Cyan	0.311
Black	0.589
Red	0.556
White	0.700
Green	0.567
Grey	0.534

Step 7 - Rank Alternatives

Alternative	C-Rank
Orange	7
Violet	9
Blue	6
Yellow	8
Cyan	10
Black	2
Red	4
White	1
Green	3
Grey	5

2D Example

Seven Steps using OSD-DA

Step 1 - Collect Measures

Alternative	C1	C2
Orange	1	8
Violet	2	5
Blue	3	8
Yellow	4	4
Cyan	4	2
Black	5	7
Red	6	5
White	7	7
Green	7	4
Grey	8	2

Step 2 - Level Scales

Alternative	C1	C2
Orange	0.1	0.8
Violet	0.2	0.5
Blue	0.3	0.8
Yellow	0.4	0.4
Cyan	0.4	0.2
Black	0.5	0.7
Red	0.6	0.5
White	0.7	0.7
Green	0.7	0.4
Grey	0.8	0.2

Step 3a - Choose Weights

OSD-DA Raw Weights

Alternative	W1	W2	Average
Orange	0.000	1.250	0.650
Violet	0.000	2.000	1.040
Blue	0.000	1.250	0.650
Yellow	2.500	0.000	1.175
Cyan	2.500	0.000	1.175
Black	0.000	1.429	0.743
Red	1.667	0.000	0.783
White	1.429	0.000	0.671
Green	1.429	0.000	0.671
Grey	1.250	0.000	0.588

Step 3b - Choose Weights

OSD-DA Normalized Weights

Alternative	W1	W2
Orange	0	1
Violet	0	1
Blue	0	1
Yellow	1	0
Cyan	1	0
Black	0	1
Red	1	0
White	1	0
Green	1	0
Grey	1	0

Step 4 - Stack Weights

OSD-IP Stacked Weights

6	4

Step 5 - Calculate WIF

OSD-IP Normalized Weights

60%	40%

Step 6 - Calculate Scores

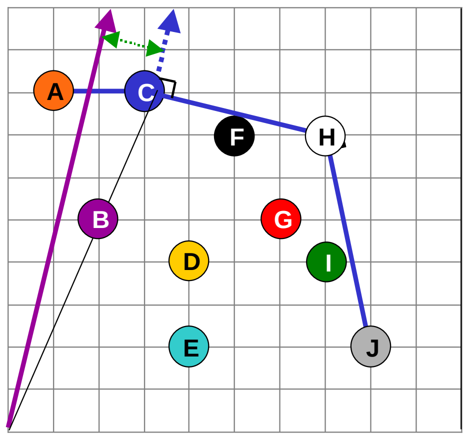
Alternative	C-Score
Orange	0.380
Violet	0.320
Blue	0.500
Yellow	0.400
Cyan	0.320
Black	0.580
Red	0.560
White	0.700
Green	0.580
Grey	0.560

Step 7 - Rank Alternatives

Alternative	C-Rank
Orange	8
Violet	9
Blue	6
Yellow	7
Cyan	9
Black	3
Red	4
White	1
Green	2
Grey	4

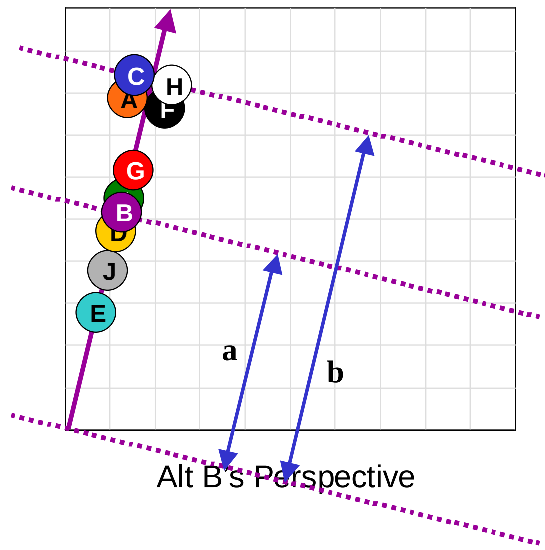
OSD-CCR Results Alt B

Frontier defined by those who can rank first



Projection & Weight Selection for Alt B

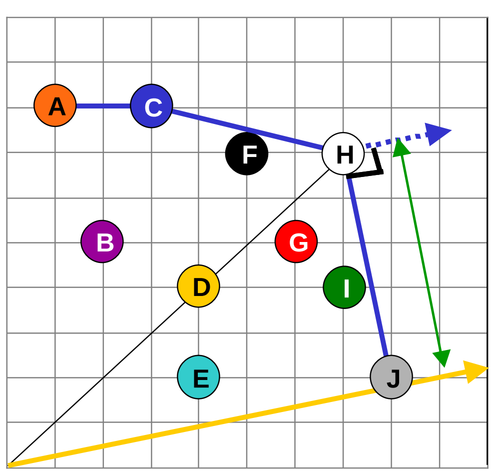
Maximize Score = a/b



Alt B's Perspective

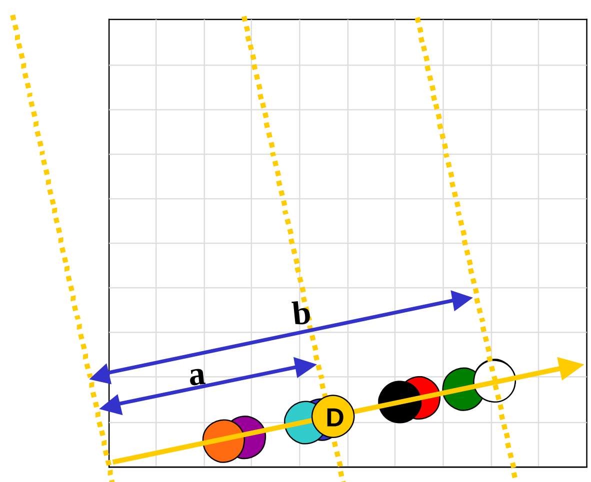
OSD-CCR Results Alt D

Frontier defined by those who can rank first



Projection & Weight Selection for Alt D

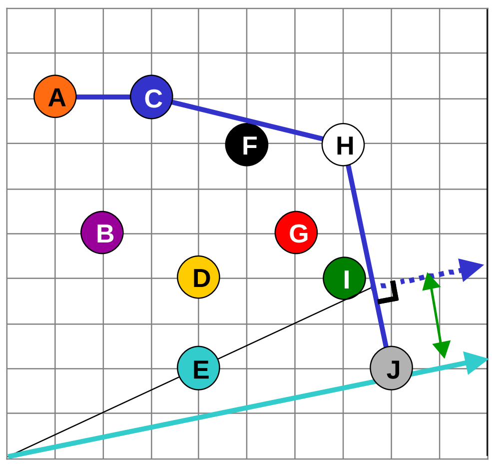
Maximize Score = a/b



Alt D's Perspective

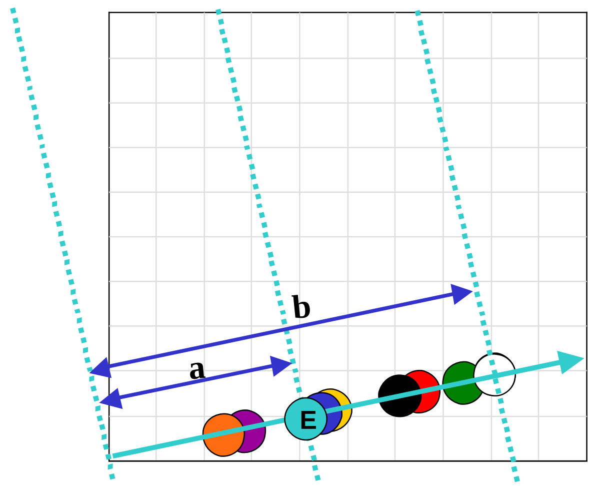
OSD-CCR Results Alt E

Frontier defined by those who can rank first



Projection & Weight Selection for Alt E

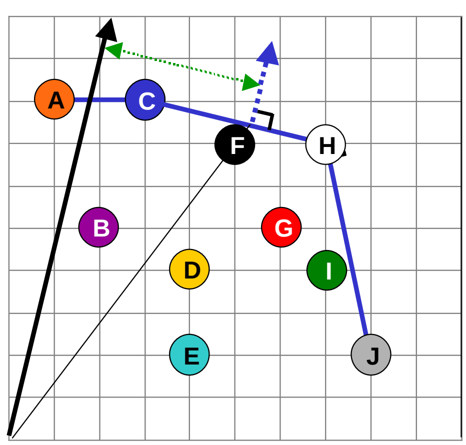
Maximize Score = a/b



Alt E's Perspective

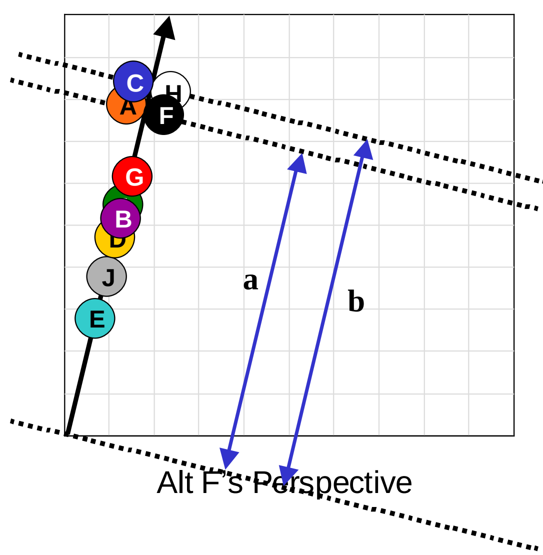
OSD-CCR Results Alt F

Frontier defined by those who can rank first



Projection & Weight Selection for Alt F

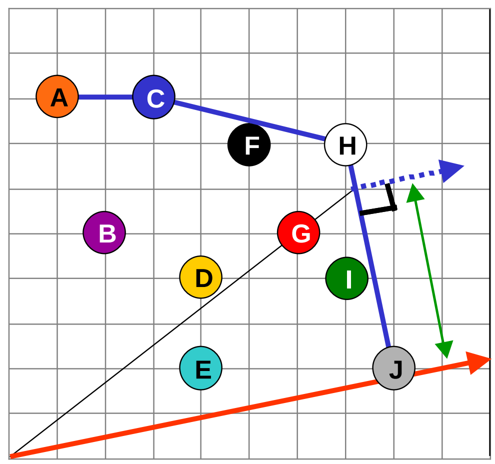
Maximize Score = a/b



Alt F's Perspective

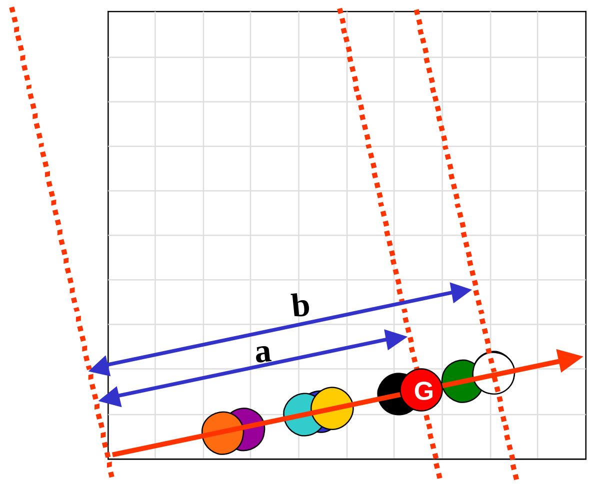
OSD-CCR Results Alt G

Frontier defined by those who can rank first



Projection & Weight Selection for Alt G

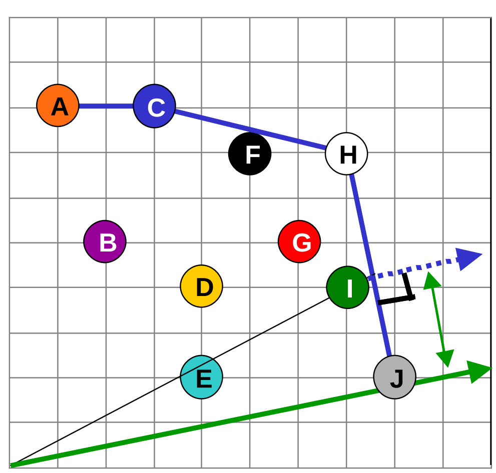
Maximize Score = a/b



Alt G’s Perspective

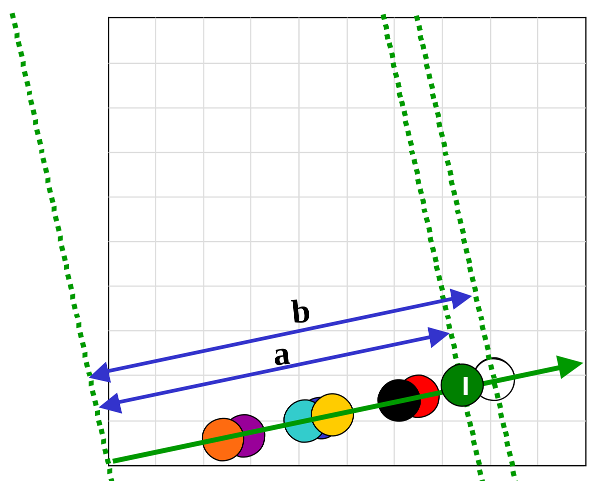
OSD-CCR Results Alt I

Frontier defined by those who can rank first



Projection & Weight Selection for Alt I

Maximize Score = a/b

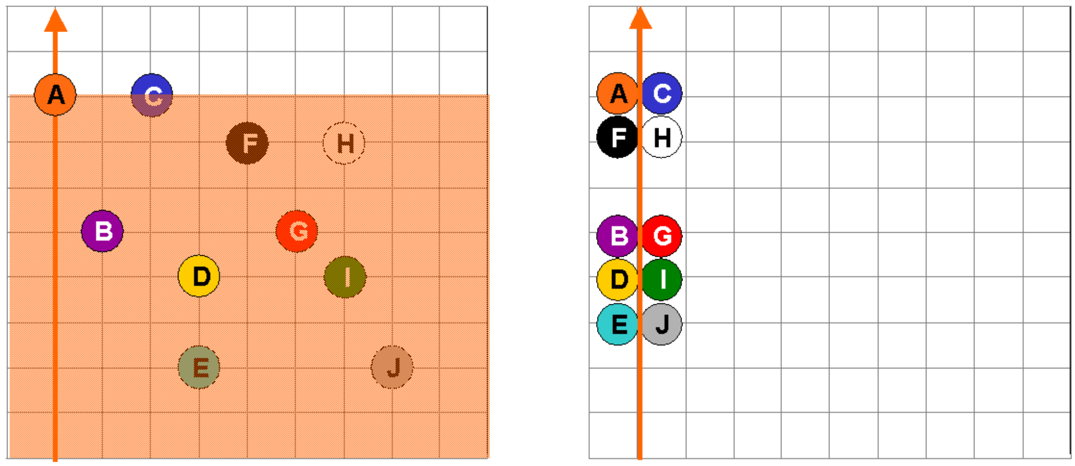


Alt I’s Perspective

OSD-IP Results Alt A

Choose Weights to Maximize Rank

Number Above = 0



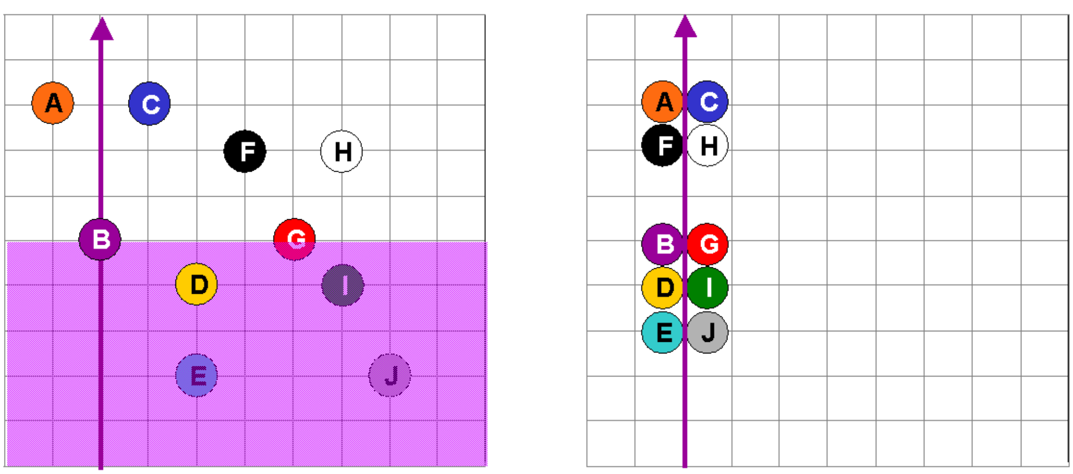
Solution for DMU A

Max Rank = 1

OSD-IP Results Alt B

Choose Weights to Maximize Rank

Number Above = 4

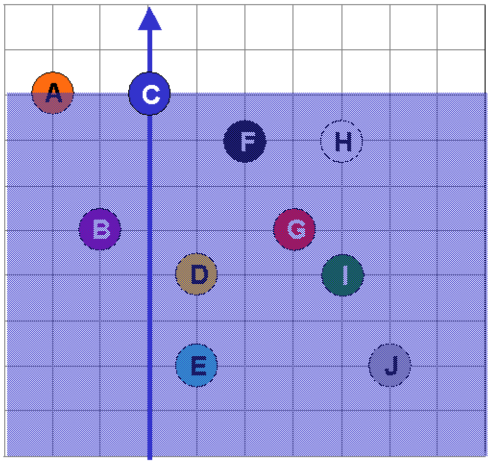


Solution for DMU B

Max Rank = 5

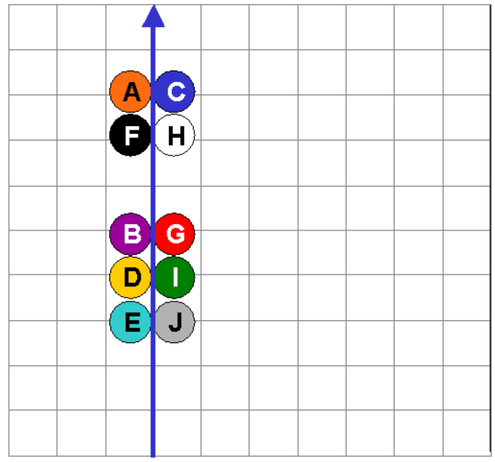
OSD-IP Results Alt C

Choose Weights to Maximize Rank



Solution for DMU C

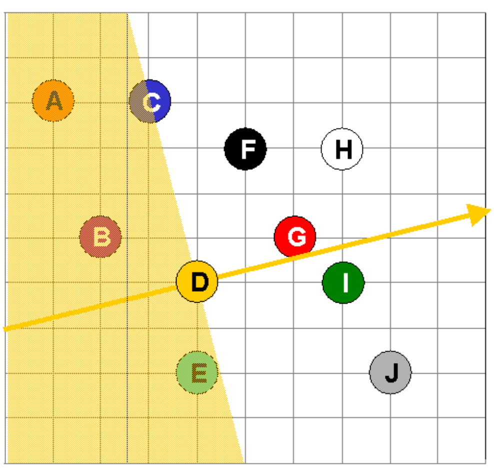
Number Above = 0



Max Rank = 1

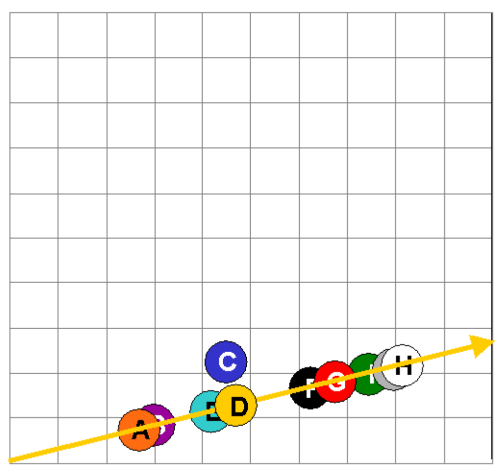
OSD-IP Results Alt D

Choose Weights to Maximize Rank



Solution for DMU D

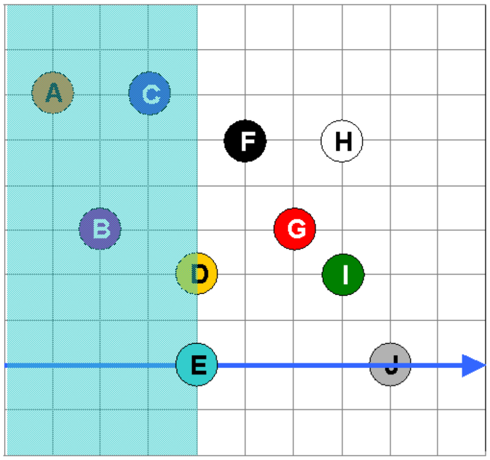
Number Above = 5



Max Rank = 6

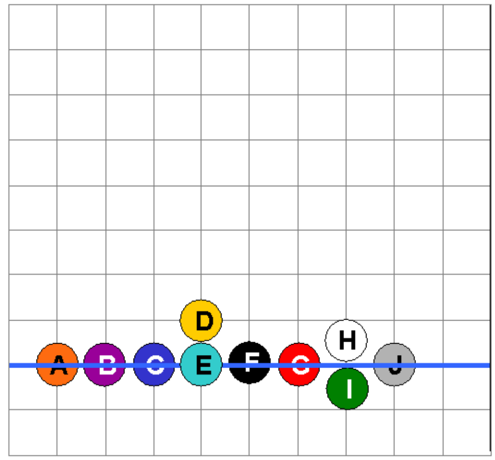
OSD-IP Results Alt E

Choose Weights to Maximize Rank



Solution for DMU E

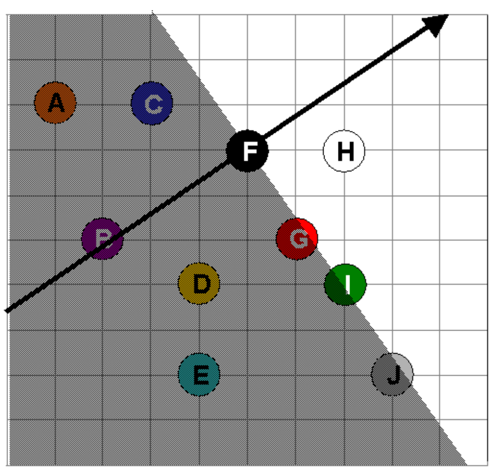
Number Above = 5



Max Rank = 6

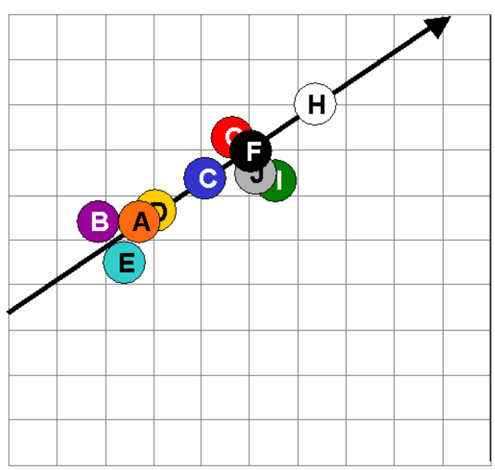
OSD-IP Results Alt F

Choose Weights to Maximize Rank



Solution for DMU F

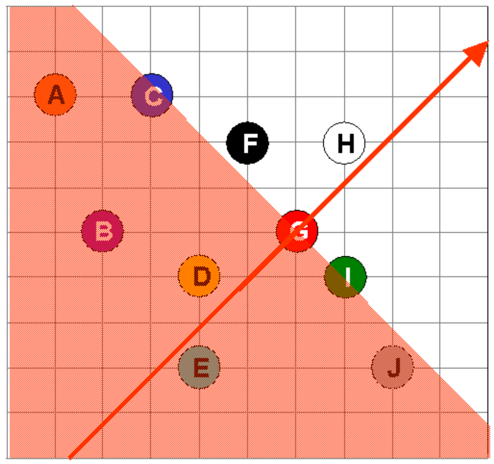
Number Above = 1



Max Rank = 2

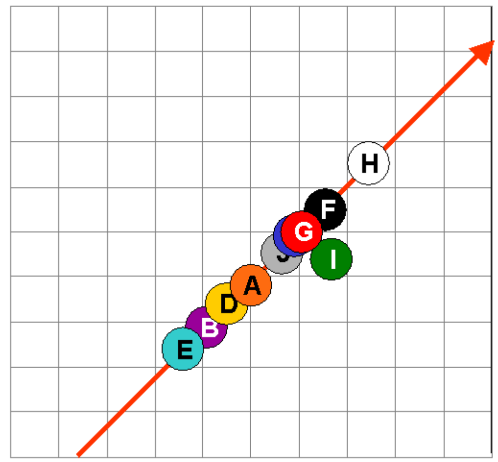
OSD-IP Results Alt G

Choose Weights to Maximize Rank



Solution for DMU G

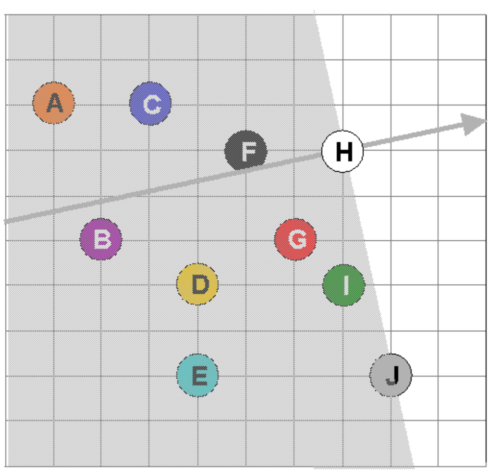
Number Above = 2



Max Rank = 3

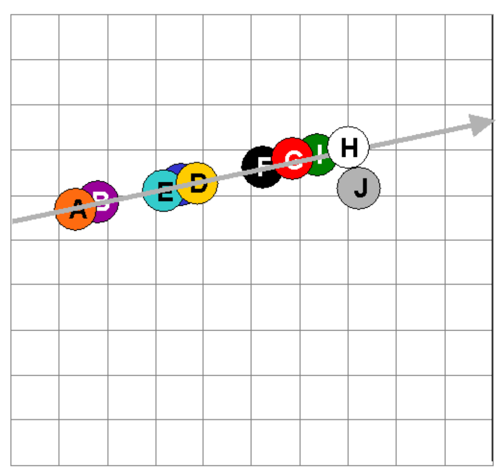
OSD-IP Results Alt H

Choose Weights to Maximize Rank



Solution for DMU H

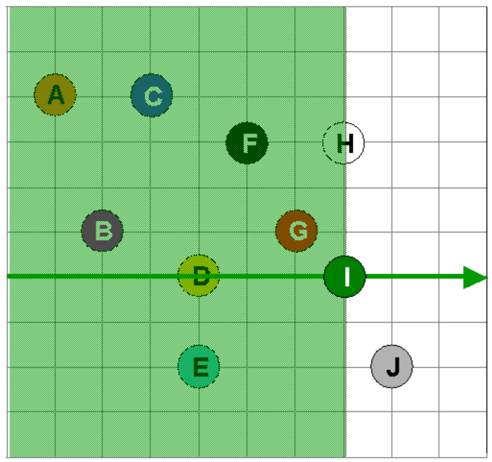
Number Above = 0



Max Rank = 1

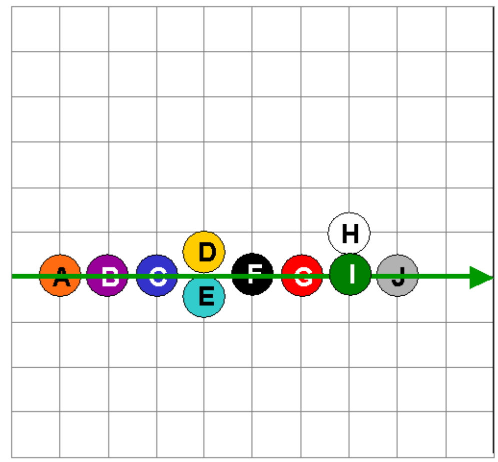
OSD-IP Results Alt I

Choose Weights to Maximize Rank



Solution for DMU I

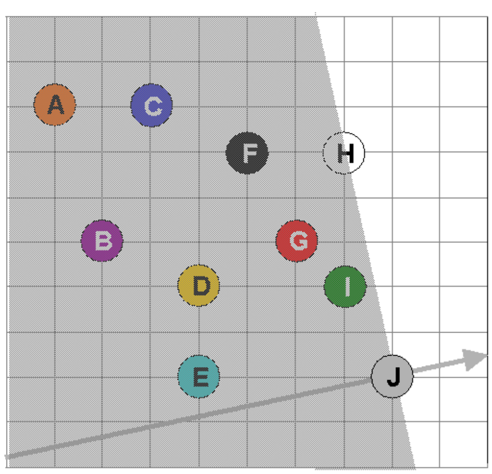
Number Above = 1



Max Rank = 2

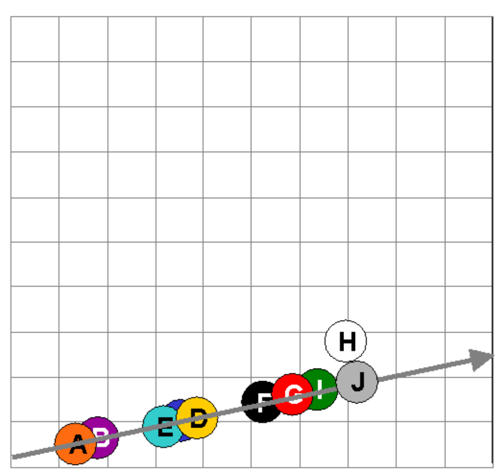
OSD-IP Results Alt J

Choose Weights to Maximize Rank



Solution for DMU J

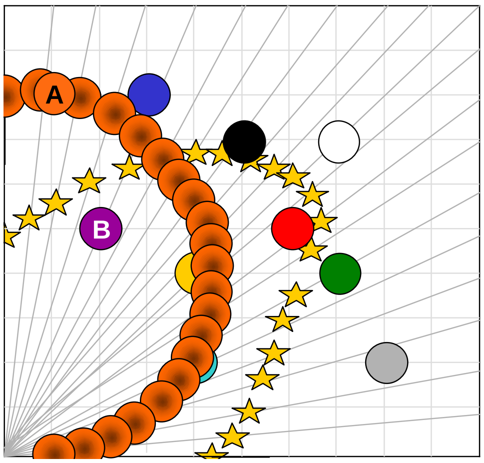
Number Above = 0



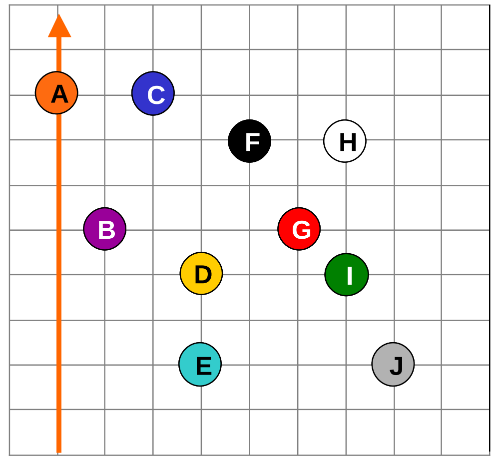
Max Rank = 1

OSD-DA Results Alt A

Distinct from Average



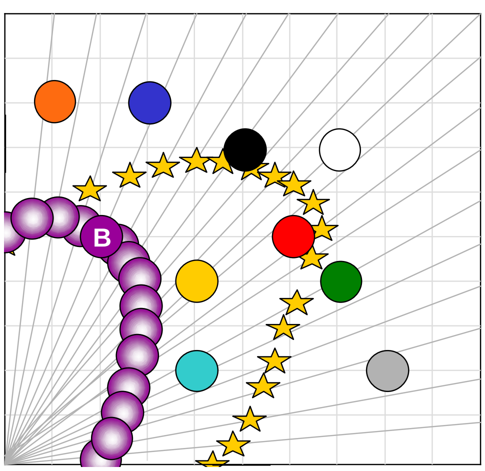
Optimal direction for Alt-A



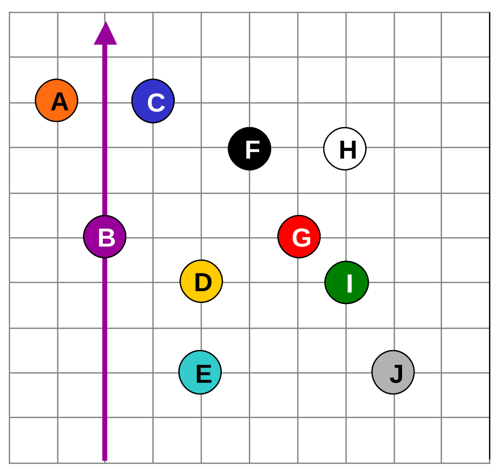
Solution: Stars indicate average for each direction

OSD-DA Results Alt B

Distinct from Average



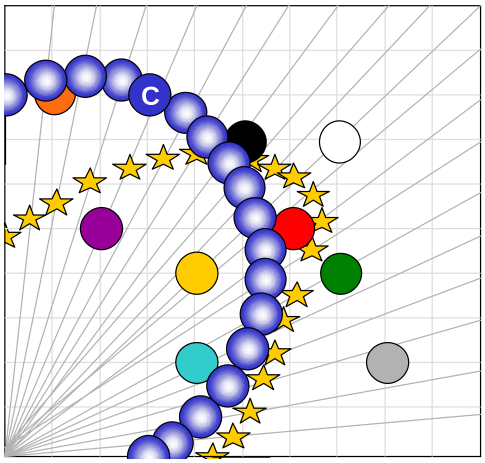
Optimal direction for Alt-G



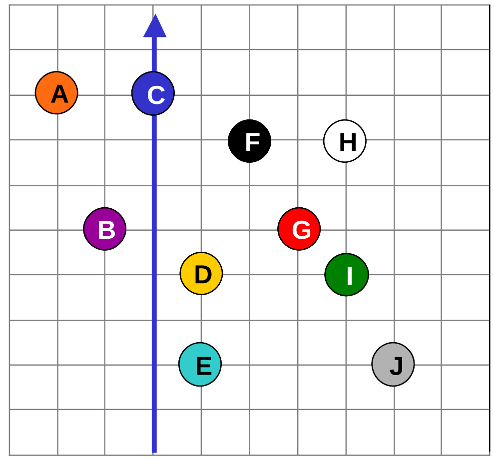
Solution: Stars indicate average for each direction

OSD-DA Results Alt C

Distinct from Average



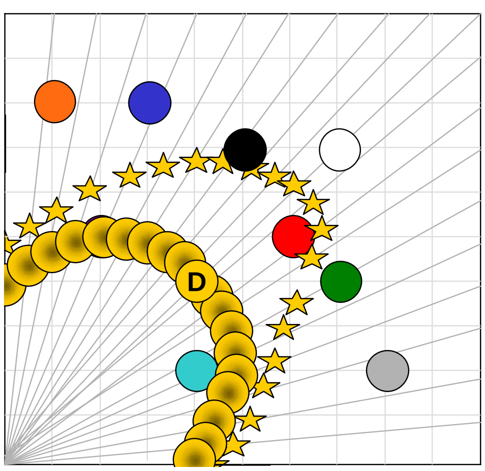
Optimal direction for Alt-C



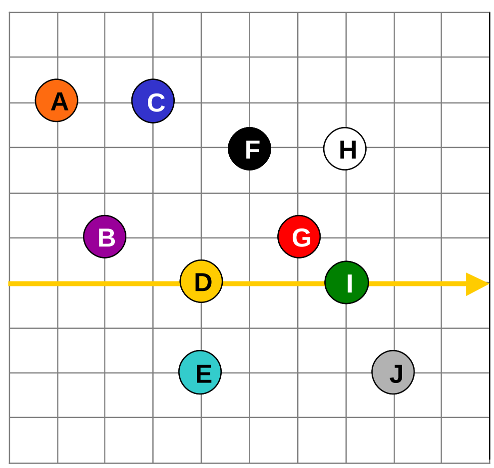
Solution: Stars indicate average for each direction

OSD-DA Results Alt D

Distinct from Average



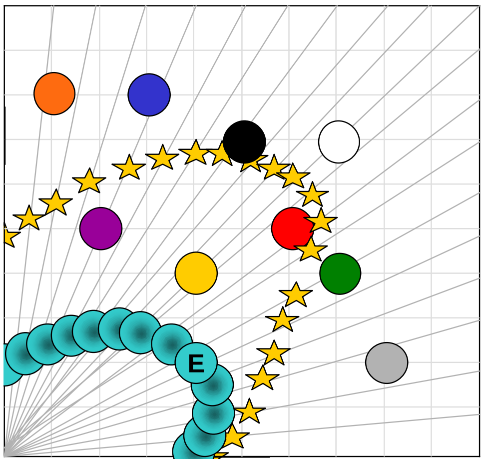
Optimal direction for Alt-D



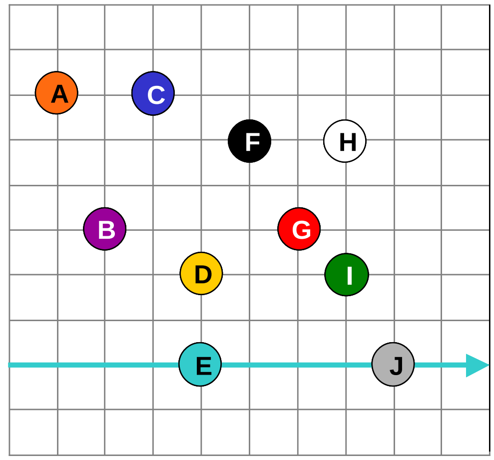
Solution: Stars indicate average for each direction

OSD-DA Results Alt E

Distinct from Average



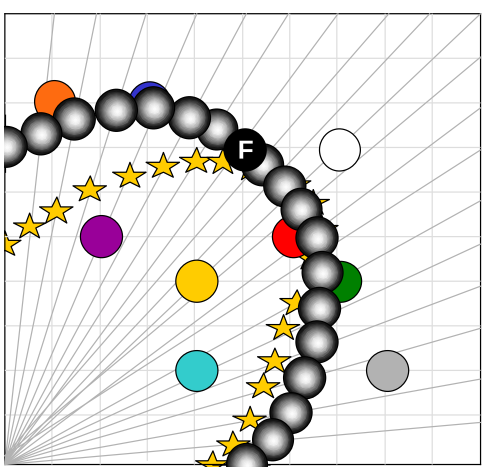
Optimal direction for Alt-E



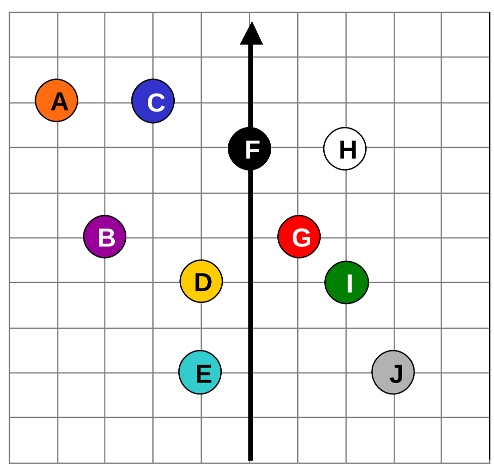
Solution: Stars indicate average for each direction

OSD-DA Results Alt F

Distinct from Average

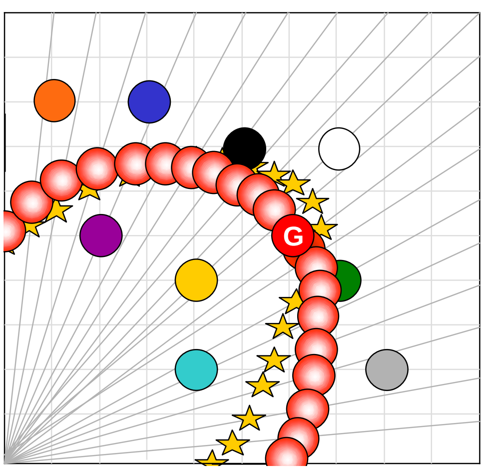


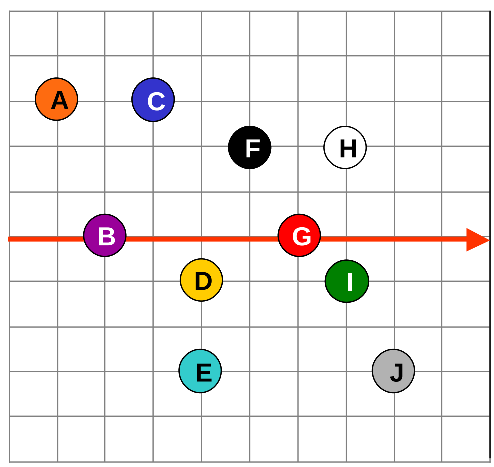
Optimal direction for Alt-F



Solution: Stars indicate average for each direction

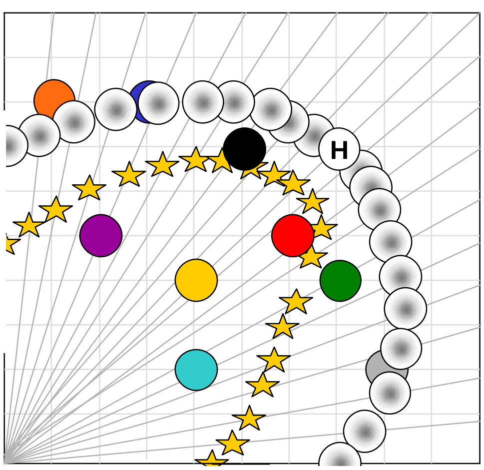
OSD-DA Results Alt G

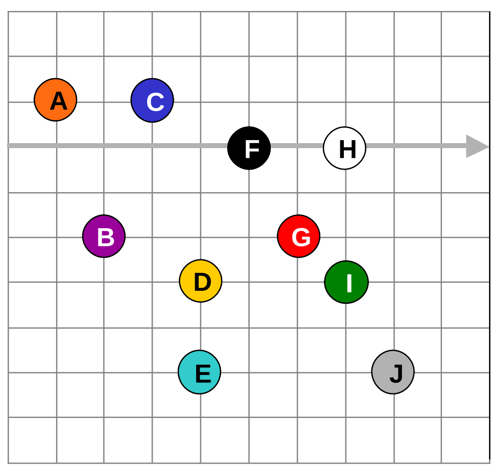




Solution: Stars indicate average for each direction

OSD-DA Results Alt H

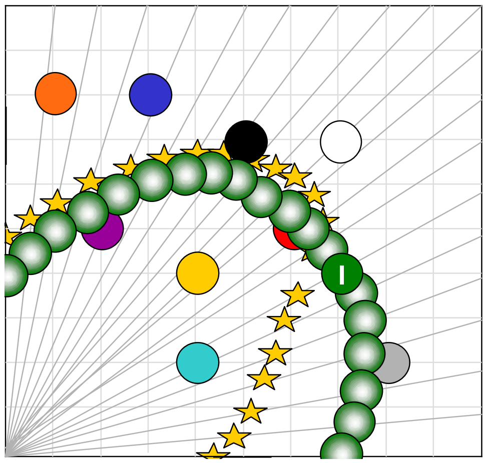




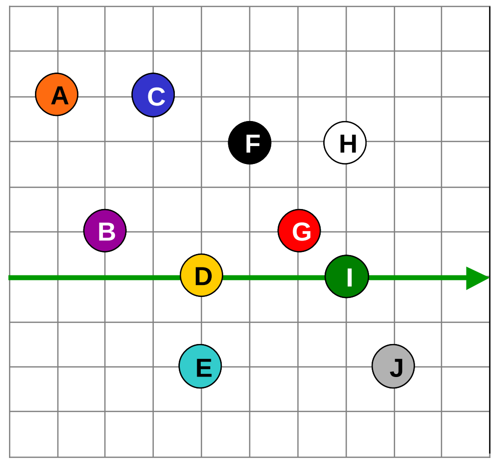
Solution: Stars indicate average for each direction

OSD-DA Results Alt I

Distinct from Average



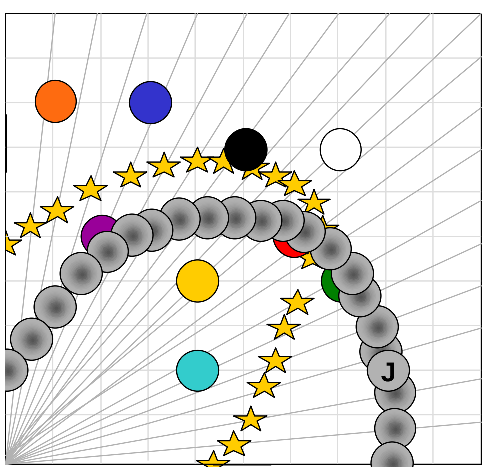
Optimal direction for Alt-I



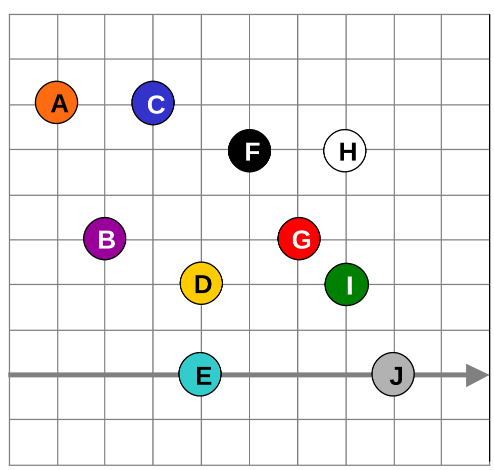
Solution: Stars indicate average for each direction

OSD-DA Results Alt J

Distinct from Average

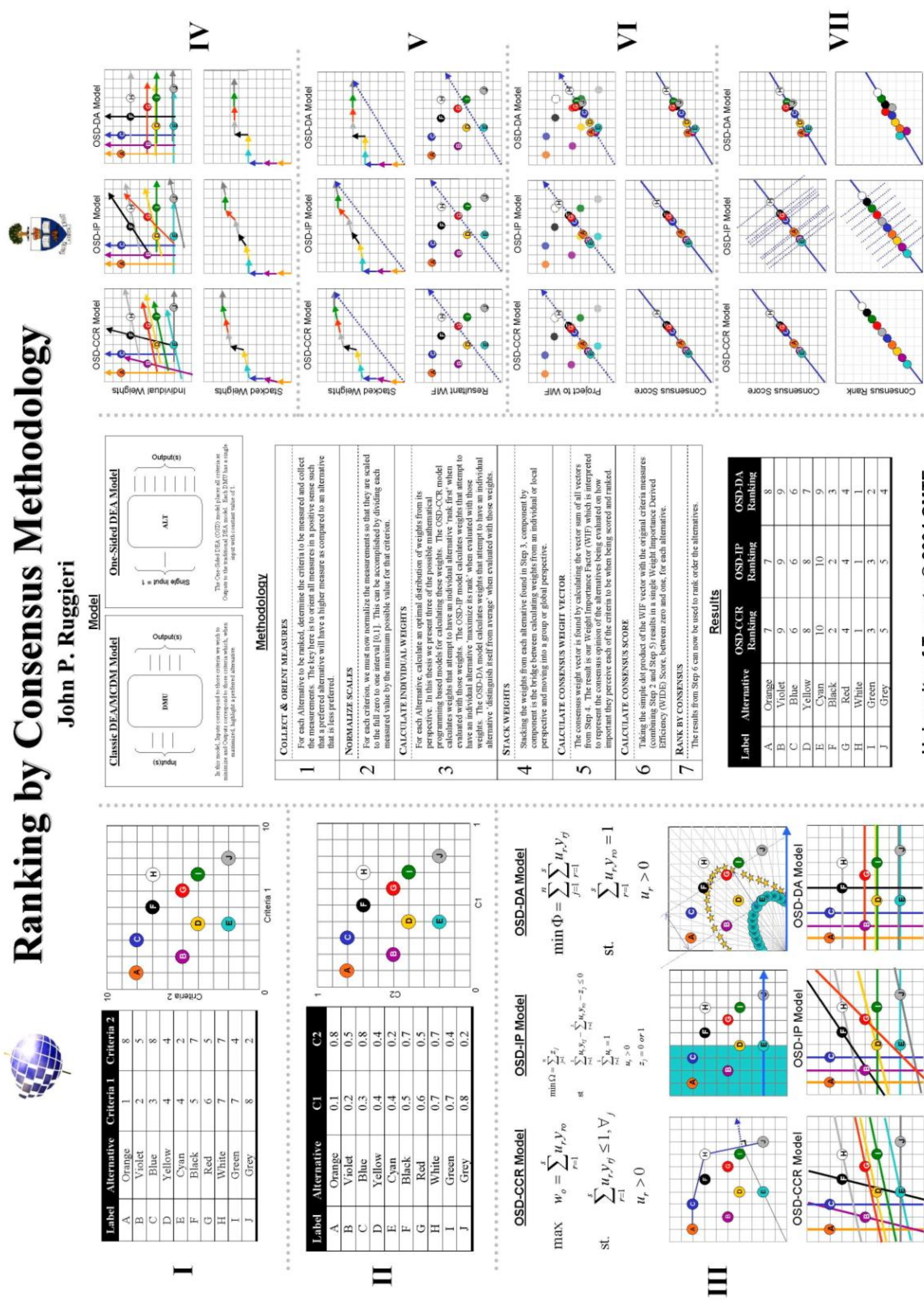


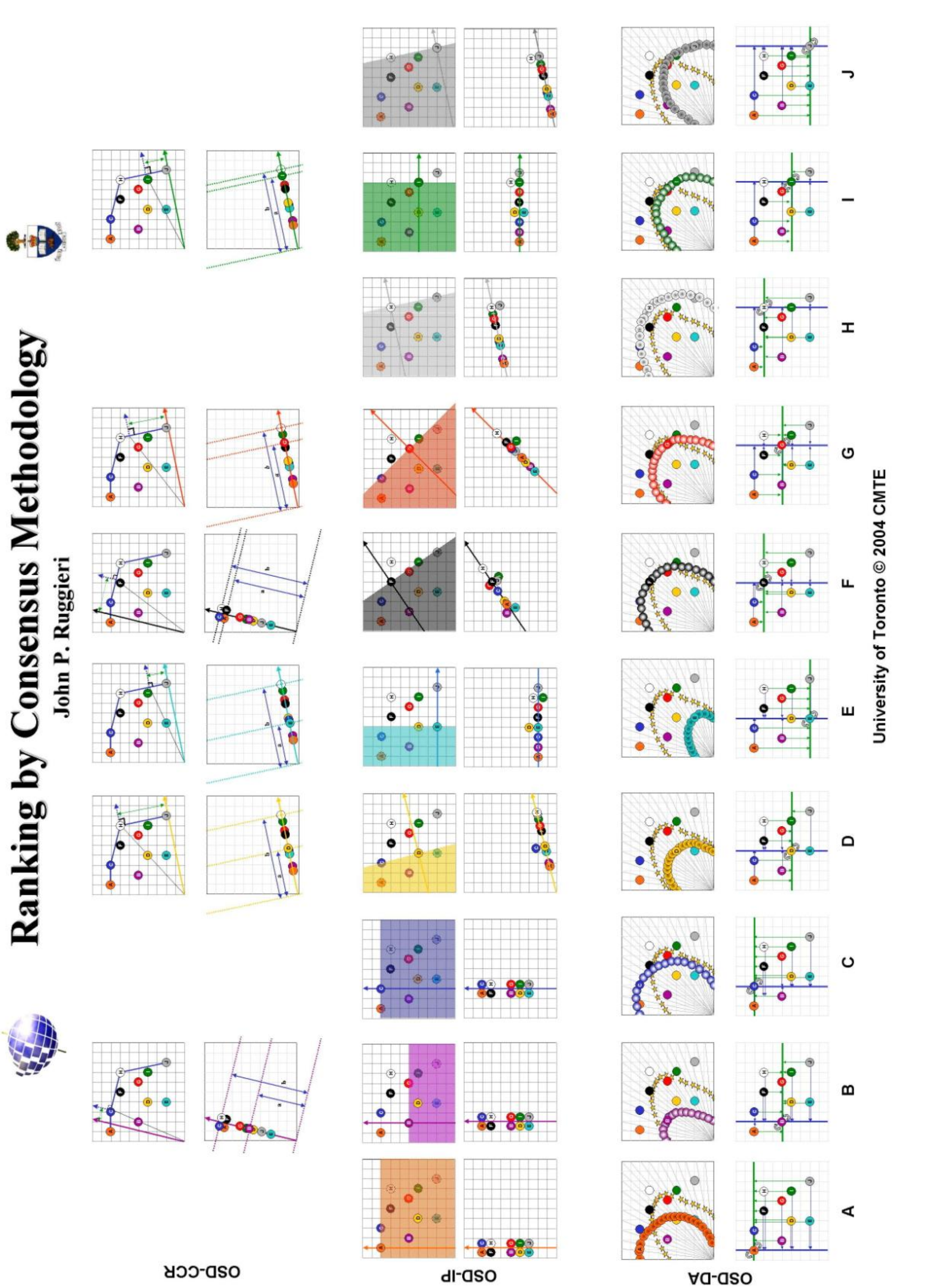
Optimal direction for Alt-J



Solution: Stars indicate average for each direction

Appendix E – RCM Poster Presentation




Ranking by Consensus Methodology
John P. Ruggieri
OSD-CCR
OSD-IP
OSD-DA
A
B
C
D
E
F
G
H
I
J
University of Toronto © 2004 CMTE